1. Let $\mathbb{T}$ denote the Cauchy stress tensor in $\mathbb{R}^2$, $\mathbb{T} = \begin{bmatrix} \mathrm{T}_{\hat{x}\hat{x}} & \mathrm{T}_{\hat{x}\hat{y}} \\ \mathrm{T}_{\hat{y}\hat{x}} & \mathrm{T}_{\hat{y}\hat{y}} \end{bmatrix}$, and let $\boldsymbol{n}$ denote the normal to a given surface element, and let $\boldsymbol{t}$ be any vector such that $\boldsymbol{t} \bullet \boldsymbol{n} = 0$. If the surface element is oriented in such a way that $\mathbb{T}\boldsymbol{n} = \tau\boldsymbol{n}$, where τ is a number, then we say that the surface element with the orientation $\boldsymbol{n}$ experiences a *pure tension/compression.* (The direction of *traction* is parallel to the normal to the surface element.) If the surface element is oriented in such a way that $\mathbb{T}\boldsymbol{n} \bullet \boldsymbol{t} \neq \boldsymbol{0}$, then we say that the surface element with the orientation $\boldsymbol{n}$ experiences a *shear stress.*

   - Show that it is always possible to find a surface element with normal $\boldsymbol{n}$ such that the element experiences the *pure tension/compression.*
   - Find the orientation $\boldsymbol{n}$ of the surface that is subject to the *maximal shear stress.* In other words, what is the value of $\boldsymbol{n}$ that maximises
   $$\left|(\mathbb{I} - \boldsymbol{n} \otimes \boldsymbol{n})\, \mathbb{T}\boldsymbol{n}\right| = \left|\mathbb{T}\boldsymbol{n} - ((\mathbb{T}\boldsymbol{n}) \bullet \boldsymbol{n})\, \boldsymbol{n}\right|.$$
   (Try to guess what is the solution before you make the formal computation.)
   - What is the relation between the maximal achievable tension/compression and the maximal achievable shear stress? (Try to guess what is the solution before you make the formal computation.)

   It would be nice to get the answers in a coordinate free form, let us say by referring only to the eigenvalues and eigenvectors of the Cauchy stress tensor $\mathbb{T}$.