1. Let $\boldsymbol{v}$ denote the Eulerian velocity field. We already know that

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\partial \boldsymbol{v}}{\partial t} + (\operatorname{rot} \boldsymbol{v}) \times \boldsymbol{v} + \nabla \left(\frac{1}{2} \boldsymbol{v} \bullet \boldsymbol{v} \right),$$

where $\frac{\mathrm{d}}{\mathrm{d}t}$ is the material time derivative. Show that

$$\frac{\mathrm{d}}{\mathrm{d}t} (\operatorname{rot} \boldsymbol{v}) = \operatorname{rot} \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + ((\operatorname{rot} \boldsymbol{v}) \bullet \nabla) \boldsymbol{v} - (\operatorname{rot} \boldsymbol{v}) \operatorname{div} \boldsymbol{v}.$$