Bounds of Chord Power Integrals for Some Classes of Convex Bodies and a Remark on Determinantal Point Processes

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Abstract

First we introduce different representations of chord power integrals (briefly: CPIs) $\mathcal{I}_p(K)$ of any order $p \geq 0$ for convex bodies $K \subset \mathbb{R}^d$ with inner points. Second, we motivate the talk by discussing some situations, where CPIs occur, for example, as energy functionals in physical contexts or as asymptotic variances in CLTs for motion-invariant Poisson cylinder and hyperplane processes.

Third, we prove specific representations and lower as well upper bounds of $\mathcal{I}_p(K)$ $(1 \leq p \leq d)$ for particular convex bodies such as ellipsoids, parallelotopes and hyperrectangles. We show that the pth-order CPI of parallelotopes with fixed volume is maximized by the pth-order CPI of the d-cube with the same volume. On the other hand, among all parallelotopes with fixed total edge length the d-cube possesses the least pth-order CPI with the same total edge length. Analogous extremal properties could be obtained also for ellipsoids $\mathbb{E}(\mathbf{a})$ with semi-axes $\mathbf{a} = (a_1, \ldots, a_d)$, see [2]. The corresponding inequalities are based on a new closed-term formula for $\mathcal{I}_p(\mathbb{E}(\mathbf{a}))$ in terms of the support function of $\mathbb{E}(\mathbf{a})$. The mentioned bounds are consistent with a longstanding (still unproved) conjecture on CPIs of general convex bodies, see [1]. Some remarks on CPIs of superellipsoids and simplices round off the topic.

Finally, we consider stationary determinantal point processes which are completely determined by a single non-negative definite function $c(\cdot)$ on $\mathbb{R}^d$. We prove the "strong Brillinger-mixing property" for this type of point processes if $c(\cdot)$ is absolutely and square integrable, see [3].

References

[1] Davy, P. J. (1984). Inequalities for moments of secant length, *Z. Wahrscheinlichkeitstheorie verw. Geb.* **68**, 243–246.

[2] Heinrich, L. (2014) Lower and upper bounds for chord power integrals of ellipsoids, *Appl. Math. Sciences* **8** (165), 8257–8269.

[3] Heinrich, L. (2015) Some remarks on determinantal point processes (paper in preparation), Talk to be delivered at the 18th Workshop SGSIA in Lingen, 22 - 27 March 2015.