Iterated Function Systems, Multifunctions and Multimeasures: Inverse Problems and Applications

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The landmark papers by Hutchinson and Barnsley and Demko showed how systems of contractive maps with associated probabilities (called "iterated function systems" by the latter), acting in a parallel manner, either deterministically or probabilistically, could be used to construct fractal sets and measures.
There is an ongoing research programme (see links.uwaterloo.ca) on the construction of appropriate IFS-type operators, or generalized fractal transforms (GFT), over various spaces, i.e., function spaces and distributions, vector-valued measures, integral transforms and wavelet transforms. The action of a GFT on an element u of the complete metric space (X,d) under consideration can be summarized as follows:
(i) it produces a set of N spatially-contracted copies of u,
(ii) it then modifies the values of these copies by means of a suitable range-mapping,
(iii) it recombines these copies using an appropriate operator to produce the element v in X, $v = Tu$.
In each of the above-mentioned cases, the fractal transform T is guaranteed to be contractive when the parameters defining it satisfy appropriate conditions specific to the metric space of concern. In this situation, Banach's fixed point theorem guarantees the existence of a unique fixed point $u = Tu$.
The inverse problem of fractal-based approximation is as follows: given an element y, can we find a fractal transform T with fixed point u so that $d(y,u)$ is sufficiently small. However, the search for such transforms is enormously complicated. Thanks to a simple consequence of Banach's fixed point theorem known as the Collage Theorem, most practical methods of solving the inverse problem seek to find an operator T for which the collage distance $d(u,Tu)$ is as small as possible.
The aim of this talk is to present some recent developments and extensions of fractal transforms and show interesting applications in image processing and economics.

Recent references:

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