A1

$$\lim_{n\to\infty} \frac{(n+1)^n + n! + 3^n \cdot \sin n}{(-1)^n \cdot n^7 + n^n} =$$

$$= \lim_{n\to\infty} \frac{(n+1)^n \left(1 + \frac{n!}{(n+1)^n} + \frac{3^n}{(n+1)^n} \cdot \sin n\right)}{n^n \left(\frac{(-1)^n \cdot n^7}{n^n} + 1\right)} =$$

$$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n \cdot \lim \frac{1 + \frac{n!}{(n+1)^n} + \frac{3^n}{(n+1)^n} \cdot \sin n}{\frac{(-1)^n \cdot n^7}{n^n} + 1} =$$

- $\lim \frac{3^n}{(n+1)^n} = 0$, takže $\lim \underbrace{\frac{3^n}{(n+1)^n}}_{\text{nulová}} \cdot \underbrace{\sin n}_{\text{omez}} = 0$

- $\lim \frac{(-1)^n \cdot n^7}{n^n} = \lim \underbrace{\frac{1}{n^{n-7}}}_{\text{nulová}} \cdot \underbrace{(-1)^n}_{\text{omezená}} = 0$

- $\frac{n!}{(n+1)^n} \leq \frac{n!}{n^n} \longrightarrow 0.$

$$= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n \cdot \frac{1+0+0}{0+1} = e$$

$\boxed{A2}$

$$\lim_{n\to\infty} \sqrt[n]{n^7+6^n+5^5+400\cdot 4^n} \cdot \frac{\frac{(n+2)!}{n!}-n^2}{n} =$$

$$\left[\frac{\frac{(n+2)!}{n!}-n^2}{n} = \frac{(n+2)(n+1)-n^2}{n} = \frac{3n+2}{n} \to 3 \right]$$

$$= \lim \frac{3n+2}{n} \cdot \lim \sqrt[n]{n^7+6^n+5^5+400\cdot 4^n} =$$

$$3 \cdot \lim_{n\to\infty} \sqrt[n]{n^7+6^n+5^5+400\cdot 4^n} = \underline{\underline{3\cdot 6}}$$

$$\left[6 = \sqrt[n]{6^n} \leq \sqrt[n]{n^7+6^n+5^5+400\cdot 4^n} \leq \sqrt[n]{402\cdot 6^n} \right.$$

$$\left. \begin{array}{ccc} \downarrow & \text{Podle LOZP:} \downarrow & \begin{array}{c} \| \\ \sqrt[n]{402}\cdot\sqrt[n]{6^n} \\ \| \\ \sqrt[n]{402}\cdot 6 \\ \downarrow \\ 1\cdot 6 \end{array} \\ 6 & 6 & \end{array} \right]$$

(8)

A3

$$\lim_{n\to\infty} \left(\frac{n^{18} + 1000\, n^{17}}{2n^{18} + 1000\, n^{19}} \right)^{20} \cdot \frac{n!}{9^n} =$$

$$= \lim_{n\to\infty} \left(\frac{n^{18} \cdot \left(1 + \frac{1000}{n}\right)}{n^{19} \left(\frac{2}{n} + 1000 \right)} \right)^{20} \cdot \frac{n!}{9^n} =$$

$$= \left(\lim \frac{1 + \frac{1000}{n}}{\frac{2}{n} + 1000} \right)^{20} \cdot \lim \frac{1}{n^{20}} \cdot \frac{n!}{9^n} =$$

$$= \left(\frac{1}{1000} \right)^{20} \cdot \lim_{n\to\infty} \underbrace{\frac{2^n}{n^{20}}}_{\to\infty} \cdot \underbrace{\frac{n!}{18^n}}_{\to\infty} = \infty \cdot \infty = \infty$$

podle SROV. ŠKÁLY

B1

$$\lim_{n\to\infty} \sqrt[n]{\pi^n + e^n + \left(\tfrac{22}{7}\right)^n + 100n^{100}}$$ – označme a_n

Určíme převladající člen. $\pi > e$, $\pi \doteq 3{,}14159$.

$\frac{22}{7} = ?$ $\quad 22 : 7 = 3{,}14285\ldots \Rightarrow \frac{22}{7} > \pi$.

10
30
20
60
40

Podle měřítkové škály $(22/7)^n \succ 100n^{100}$, tj.

$$\lim \frac{100n^{100}}{\left(\frac{22}{7}\right)^n} = 0,$$ tj. tedy $\exists n_0\ \forall n \geq n_0 : 100n^{100} < (22/7)^n$

Tedy:

$$\sqrt[n]{\left(\tfrac{22}{7}\right)^n} \leq a_n \overset{n\geq n_0}{\leq} \sqrt[n]{\left(\tfrac{22}{7}\right)^n + \left(\tfrac{22}{7}\right)^n + \left(\tfrac{22}{7}\right)^n + \left(\tfrac{22}{7}\right)^n}$$

$$\sqrt[n]{\left(\tfrac{22}{7}\right)^n} = \frac{22}{7} \to \frac{22}{7}$$

$$\sqrt[n]{\left(\tfrac{22}{7}\right)^n + \left(\tfrac{22}{7}\right)^n + \left(\tfrac{22}{7}\right)^n + \left(\tfrac{22}{7}\right)^n} = \sqrt[n]{4\cdot\left(\tfrac{22}{7}\right)^n} = \sqrt[n]{4}\cdot\frac{22}{7} \to \frac{22}{7}$$

$$\Rightarrow \quad a_n \to \frac{22}{7} \quad \Leftarrow$$

Celkem: $\lim\limits_{n\to\infty} a_n = \frac{22}{7}$

$\boxed{B2}$

$$\lim_{n\to\infty} \frac{(2n)! - 10^{4n} + n^n\cdot\cos n}{3\cdot n^n + 5\cdot 100^{2n} - 2\cdot n!} =$$

$$\left[\ \lim \frac{n^n}{(2n)!} = 0 \ :\ 0 \le \frac{n^n}{2n!} = \frac{n\cdot n\cdot\ldots\cdot n}{2n\cdot(2n-1)\cdot\ldots\cdot n}\cdot\frac{1}{(n-1)!} < \frac{1}{(n-1)!} \to 0\ \right.$$

Podle LOZP to dostaneme.

$$= \lim_{n\to\infty} \frac{(2n)!\left(1 - \frac{10^{4n}}{(2n)!} + \underbrace{\frac{n^n}{(2n!)}}_{\text{„nulová"}}\cdot\underbrace{\cos n}_{\text{omez.}}\right)}{n^n\left(3 + \frac{5\cdot 100^{2n}}{n^n} - 2\frac{n!}{n^n}\right)} =$$

„nulová" · omez. $\to 0$

$$\frac{1-0+0}{3+0-0}\cdot\lim_{n\to\infty}\frac{(2n)!}{n^n} = \frac{1}{3}\cdot\infty =$$

$$= \underline{\underline{\infty}}$$

$\boxed{B3}$

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+2n^2}-\sqrt[3]{n^3+3n}\right)\cdot\left(\frac{3n^3+2n^2-n}{n+2n^3}\right)^3 =$$

$$=\left(\lim \frac{n^3\cdot\left(3+\frac{2}{n}-\frac{1}{n^2}\right)}{n^3\left(2+\frac{1}{n^2}\right)}\right)^3\cdot \lim_{n\to\infty}\frac{n^3+2n^2-(n^3+3n)}{(n^3+2n^2)^{\frac{2}{3}}+(n^3+2n^2)^{\frac{1}{3}}(n^3+3n)^{\frac{1}{3}}+(n^3+3n)^{\frac{2}{3}}}$$

$$=\left(\frac{3}{2}\right)^3\cdot\lim \frac{2n^2-3n}{n^2\left(\left(1+\frac{2}{n}\right)^{\frac{2}{3}}+\left(1+\frac{2}{n}\right)^{\frac{1}{3}}\left(1+\frac{3}{n^2}\right)^{\frac{1}{3}}+\left(1+\frac{3}{n^2}\right)^{\frac{2}{3}}\right)} =$$

$$=\left(\frac{3}{2}\right)^3\cdot \lim\frac{n^2\left(2-\frac{3}{n}\right)}{n^2}\cdot\lim\frac{1}{(1+\ldots)^{\frac{2}{3}}+(\ )(\ )+(\ )} =$$

$$=\left(\frac{3}{2}\right)^3\cdot(2-0)\cdot\frac{1}{1+1\cdot1+1}=\underline{\underline{\frac{27}{4}}}$$

C1

$$\lim_{n\to\infty}\left(\sqrt{n^5+3n^2-n}-\sqrt{n^5+n}\right)\sqrt{n} =$$

$$= \lim_{n\to\infty} \frac{n^5+3n^2-n-(n^5+n)}{n^{5/2}\left(\sqrt{1+\frac{3}{n^3}-\frac{1}{n^4}}+\sqrt{1+\frac{1}{n^4}}\right)}\sqrt{n} =$$

$$= \lim_{n\to\infty} \frac{(3n^2-2n)\sqrt{n}}{n^{5/2}\left(\sqrt{1+\ldots}+\sqrt{1+\ldots}\right)} = \lim_{n\to\infty} \frac{\cancel{n^{\frac{5}{2}}}\left(3-\frac{2}{n}\right)}{\cancel{n^{\frac{5}{2}}}\left(\sqrt{1+\ldots}+\sqrt{1+\ldots}\right)} =$$

$$= \lim_{n\to\infty} \frac{3-\frac{2}{n}}{\sqrt{1+\ldots}+\sqrt{1+\ldots}} = \frac{3-0}{\sqrt{1}+\sqrt{1}} = \frac{3}{2}$$

$\boxed{C1}$

$$\lim_{n\to\infty} \frac{\sqrt[6]{7x^5+8+3x^9} - \sqrt[5]{-32x^7+243x^5}}{\sqrt[4]{x^6-1000x^3} + \sqrt{48x^3+64x^2}} =$$

$$= \lim_{n\to\infty} \frac{x^{\frac{9}{6}}\sqrt[6]{\frac{7}{x^4}+\frac{8}{x^9}+3} - x^{\frac{7}{5}}\sqrt[5]{\frac{243}{x^2}-32}}{x^{\frac{6}{4}}\sqrt[4]{1-\frac{1000}{x^3}} + x^{\frac{3}{2}}\sqrt{48+\frac{64}{x}}}$$

$$\left[\ \frac{7}{5}-\frac{3}{2} = \frac{14-15}{10} = \right.$$
$$= -\frac{1}{10}$$

$$= \lim_{n\to\infty} \frac{x^{\frac{3}{2}}\left(\sqrt[6]{\ldots+3} - \frac{1}{\sqrt[10]{x}}\cdot\sqrt[5]{\ldots}\right)}{x^{\frac{3}{2}}\left(\sqrt[4]{1-\frac{1000}{x^3}} + \sqrt{48+\frac{64}{x}}\right)} =$$

$$= \frac{\sqrt[6]{3} - 0\cdot\sqrt[5]{-32}}{\sqrt[4]{1} + \sqrt{48}} = \frac{\sqrt[6]{3}}{1+4\cdot\sqrt{3}}$$

(A)

C3

$$\lim_{n\to\infty}\left(\frac{(4n^2-4)\cdot(2n+1)!}{2n\cdot(2n+2)!}\right)^n =$$

$$= \lim_{n\to\infty}\left(\frac{(2n-2)(2n+2)}{2n\cdot(2n+2)}\right)^n = \lim_{n\to\infty}\left(\frac{2n-2}{2n}\right)^n =$$

$$= \lim\left(\frac{n-1}{n}\right)^n = \lim\left(1-\frac{1}{n}\right)^n = \underline{\underline{e^{-1}}}$$

$\boxed{D1}$

$$\lim_{n\to\infty} n^{\frac{15}{4}}\left(\sqrt[3]{n+\frac{2}{n}+\frac{1}{n^3}+\frac{6}{n^4}}-\sqrt[3]{n+\frac{2}{n}-\frac{1}{n^3}}\right)=$$

$$=\lim_{n\to\infty} n^{\frac{15}{4}}\frac{n+\frac{2}{n}+\frac{1}{n^3}+\frac{6}{n^4}-\left(n+\frac{2}{n}-\frac{1}{n^3}\right)}{\left(n+\frac{2}{n}+\frac{1}{n^3}+\frac{6}{n^4}\right)^{\frac{2}{3}}+\left(n+\frac{2}{n}+\frac{1}{n^3}+\frac{6}{n^4}\right)^{\frac{1}{3}}\left(n+\frac{2}{n}-\frac{1}{n^3}\right)^{\frac{1}{3}}+\left(n+\frac{2}{n}-\frac{1}{n^3}\right)^{\frac{2}{3}}}$$

$$=\lim_{n\to\infty} n^{\frac{15}{4}}\frac{\frac{2}{n^3}+\frac{6}{n^4}}{n^{\frac{2}{3}}\left(\left(1+\frac{2}{n^2}+\frac{1}{n^4}+\frac{6}{n^5}\right)^{\frac{2}{3}}+\left(1+\frac{2}{n^2}+\frac{1}{n^4}+\frac{6}{n^5}\right)^{\frac{1}{3}}\left(1+\frac{2}{n^2}-\frac{1}{n^4}\right)^{\frac{1}{3}}+\left(1+\frac{2}{n^2}-\frac{1}{n^4}\right)^{\frac{2}{3}}\right)}$$

$$=\lim_{n\to\infty}\frac{1}{(1+\ldots)^{\frac{2}{3}}+(1+\ldots)^{\frac{1}{3}}(1+\ldots)^{\frac{1}{3}}+(1+\ldots)^{\frac{2}{3}}}\cdot\lim_{n\to\infty} n^{\frac{15}{4}-\frac{2}{3}}\cdot\frac{1}{n^3}\left(2+\frac{6}{n}\right)=$$

$$=\frac{1}{3}\cdot\lim n^{\frac{15}{4}-\frac{2}{3}-3}\cdot\lim\left(2+\frac{6}{n}\right)=\frac{1}{3}\cdot(2+0)\cdot\lim_{n\to\infty} n^{\frac{1}{12}}=\underline{\underline{\infty}}$$

$$\left[\frac{15}{4}-\frac{2}{3}-3=\frac{45-8-36}{12}=\frac{1}{12}\right]$$

D2

$$\lim_{n\to\infty}\left(\frac{3n^6+13n^4+17n^n+1}{100+5n^3+7n^6}\right)^3\cdot\sqrt[n]{\frac{6}{n^2}} =$$

- $$\lim_{n\to\infty}\sqrt[n]{\frac{6}{n^2}} = \frac{\lim\sqrt[n]{6}}{\lim\sqrt[n]{n}\cdot\lim\sqrt[n]{n}} = \frac{1}{1\cdot 1} = 1$$

- $$\lim_{n\to\infty}\frac{3n^6+13n^4+17n^n+1}{100+5n^3+7n^6} =$$

$$= \lim_{n\to\infty}\frac{n^n\left(\frac{3n^6}{n^n}+\frac{13n^4}{n^n}+17+\frac{1}{n^n}\right)}{n^6\left(\frac{100}{n^6}+\frac{5}{n^3}+7\right)} =$$

$$= \lim_{n\to\infty}\frac{n^n}{n^6}\cdot\lim\frac{(\quad)}{(\quad)} = \infty\cdot\frac{0+0+17+0}{0+0+7} = \underline{\underline{\infty}}$$

D3

$$\lim_{x\to 1} \frac{2x^3+3x^2-3x-2}{x^3+x-x^2-1} = \lim_{x\to 1} \frac{(x-1)(2x^2+5x+2)}{(x-1)(x^2+1)} =$$

$$(2x^3+3x^2-3x-2):(x-1) = 2x^2+5x+2$$
$$-(2x^3-2x^2)$$
$$5x^2-3x-2$$
$$-(5x^2-5x)$$
$$2x-2$$

$$(x^3+x-x^2-1):(x-1) = x^2+1$$
$$-(x^3-x^2)$$
$$x-1$$

$$= \lim_{x\to 1} \frac{2x^2+5x+2}{x^2+1} = \frac{2+5+2}{1+1} = \frac{9}{2}$$