TEST A (29.11.2023)

$$\lim_{n\to\infty} \frac{n^{2n} - (n!)^2 + n^n \cdot \cos n}{3\cdot n^n + 5\cdot 100^{3n} - 2\cdot n!} =$$

$$n^{2n} = n^{n+n} = n^n \cdot n^n \gg n!\cdot n! = (n!)^2$$

$$= \lim \frac{n^{2n}\left(1 - \left(\frac{n!}{n^n}\right)^2 + \frac{\cos n}{n^n}\right)}{n^n\left(3 + \frac{5\cdot (10^6)^n}{n^n} - 2\frac{n!}{n^n}\right)} =$$

$$= \lim_{n\to\infty} n^n \cdot \lim_{n\to\infty} \frac{1 - \left(\frac{n!}{n^n}\right)^2 + \frac{1}{n^n}\cdot \cos n}{3 + \frac{5\cdot(10^6)^n}{n^n} - 2\cdot \frac{n!}{n^n}} =$$

$$= \infty \cdot \frac{1 - \left(\lim \frac{n!}{n^n}\right)^2 + \lim \frac{1}{n^n}\cdot \cos n}{3 + 5\cdot \lim \frac{(10^6)^n}{n^n} - 2\cdot \lim \frac{n!}{n^n}} =$$

$$= \infty \cdot \frac{1 - 0^2 + 0}{3 + 5\cdot 0 - 2\cdot 0} = \infty \cdot \frac{1}{3} = \infty .$$

Je $\lim\limits_{n\to\infty} \underbrace{\frac{1}{n^n}}_{\to} \cdot \underbrace{\cos n}_{\text{omez.}} = 0$ („nulová · omezená")

(2)
$$\lim_{n\to\infty} \sqrt[n]{\frac{6}{n^9}} \cdot \left(\frac{(n+1)^9 - (n-1)^9}{100 + 5n^3 + 3n^8}\right)^3 =$$

Dílčí úpravy: $\bullet \sqrt[n]{\frac{6}{n^9}} = \sqrt[n]{6} \cdot \left(\frac{1}{\sqrt[n]{n}}\right)^9$

$$\bullet\ (n+1)^9 - (n-1)^9 = n^9 + 9n^8 + \binom{9}{2} n^7 + \dots + 1$$
$$- \left(n^9 - 9n^8 + \binom{9}{2} n^7 - \dots - 1\right)$$
$$= 9n^8 + 9n^8 + 2\binom{9}{2} n^7 + \dots$$
$$= 18n^8 + \underbrace{A_7}\, n^7 + \underbrace{A_6}\, n^6 + \dots + \underbrace{A_1}\, n + \underbrace{A_0}.$$

nějaká čísla.

$$\overset{\text{VOAL}}{=} \lim \sqrt[n]{6} \cdot \left(\frac{1}{\lim \sqrt[n]{n}}\right)^9 \cdot \left(\lim \frac{18n^8 + A_7 n^7 + \dots}{3n^8 + 5n^3 + 100}\right)^3$$

$$= 1 \cdot \left(\frac{1}{1}\right)^9 \cdot \left(\lim_{n\to\infty} \frac{n^8\left(18 + \frac{A_7}{n} + \dots + \frac{A_0}{n^8}\right)}{n^8\left(3 + \frac{5}{n^5} + \frac{100}{n^8}\right)}\right)^3$$

$$= \left(\frac{18 + 0 + \dots + 0}{3 + 0 + 0}\right)^3 = \left(\frac{18}{3}\right)^3 = 6^3 = 216$$

(3)
$$\lim_{n\to\infty} \left(\sqrt[3]{n^3 + 2n^2} - \sqrt[3]{n^3 + 3n}\right)\left(\frac{3n^3 + 2n^2 - n}{n + 2n^3}\right)^3$$

$$= \lim_{n\to\infty} \left(\frac{n^3\left(3 + \frac{2}{n} - \frac{1}{n^2}\right)}{n^3\left(2 + \frac{1}{n^2}\right)}\right)^3 \cdot \frac{n^3 + 2n^2 - (n^3 + 3n)}{(n^3+2n^2)^{\frac{2}{3}} + (n^3+2n^2)^{\frac{1}{3}}(n^3+3n)^{\frac{1}{3}} + (n^3+3n)^{\frac{2}{3}}}$$

$$\overset{\text{VOAL}}{=} \left(\frac{3 + 0 - 0}{2 + 0}\right)^3 \cdot \lim \frac{n^2\left(2 - \frac{3}{n}\right)}{n^2\left(\left(1+\frac{2}{n}\right)^{2/3} + \left(1+\frac{2}{n}\right)^{1/3}\left(1+\frac{3}{n^2}\right)^{1/3} + \left(1+\frac{2}{n}\right)^{2/3}\right)}$$

$$(n^3 + 2n^2)^{\frac{2}{3}} = \left(n^3\cdot\left(1 + \frac{2}{n}\right)\right)^{\frac{2}{3}} = n^2 \cdot \left(1 + \frac{2}{n}\right)^{\frac{2}{3}}$$
$$(n^3 + 2n^2)^{\frac{1}{3}} = \dots = n\cdot\left(1 + \frac{2}{n}\right)^{\frac{1}{3}}$$
$$(n^3 + 3n)^{\frac{2}{3}} = \dots = n^2\left(1 + \frac{3}{n^2}\right)^{2/3}$$
$$\dots$$

$$\overset{\text{VOAL}}{=} \left(\frac{3}{2}\right)^3 \frac{2 - \lim \frac{3}{n}}{\lim\left(1 + \frac{2}{n}\right)^{2/3} + \lim\left(1 + \frac{2}{n}\right)^{1/3} \lim\left(1 + \frac{3}{n^2}\right)^{1/3} + \lim\left(1 + \frac{2}{n}\right)^{2/3}}$$

$$= \left(\frac{3}{2}\right)^3 \frac{2}{\left(\lim\left(1 + \frac{2}{n}\right)\right)^{2/3} + \dots}$$

$$= \left(\frac{3}{2}\right)^3 \cdot \frac{2}{1^{2/3} + 1^{1/3}\cdot 1^{1/3} + 1^{2/3}} = \left(\frac{3}{2}\right)^3 \cdot \frac{2}{3} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

TEST B: (29.11.2023)

(1) $$\lim_{n\to\infty} n \frac{5^n + \sqrt[n]{5} + n^7(-1)^n}{\sqrt[n]{n} - n^7 + 6^n} =$$

$$= \lim_{n\to\infty} n \cdot \frac{5^n\left(1+\frac{\sqrt[n]{5}}{5^n} + \frac{n^7}{5^n}(-1)^n\right)}{6^n\left(\frac{\sqrt[n]{n}}{6^n} - \frac{n^7}{6^n}+1\right)} =$$

$$= \lim_{n\to\infty} \frac{n\cdot 5^n}{6^n} \cdot \lim_{n\to\infty} \frac{1+\frac{\sqrt[n]{5}}{5^n} + \overbrace{\frac{n^7}{5^n}(-1)^n}^{\text{„malá · omez."}}}{\frac{\sqrt[n]{n}}{6^n} - \underbrace{\frac{n^7}{6^n}}_{\to\ \text{(„škála")}} + 1}$$

$$\overset{\text{VOAL}}{=} \lim_{n\to\infty} \frac{n}{\frac{6^n}{5^n}} \cdot \frac{1+\frac{1}{\infty}+0}{\frac{1}{\infty} - 0 + 1}$$

$$= 1\cdot \underbrace{\lim_{n\to\infty} \frac{n}{\left(\frac{6}{5}\right)^n}}_{=0\ \text{(„škála")}} = 1\cdot 0 = 0$$

(2) $$\lim_{n\to\infty} \left(\sqrt{n^5-3n^2+\sin n} - \sqrt{n^5-5n}\right)\sqrt{n}$$

$$= \lim_{n\to\infty} \frac{n^5-3n^2+\sin n-(n^5-5n)}{\sqrt{n^5-3n^2+\sin n}+\sqrt{n^5-5n}}\cdot\sqrt{n}$$

$$= \lim_{n\to\infty} \frac{(-3n^2+5n+\sin n)\sqrt{n}}{n^{\frac{5}{2}}\left(1-\frac{3}{n^3}+\frac{1}{n^5}\cdot\sin n\right)^{\frac{1}{2}} + n^{\frac{5}{2}}\left(1-\frac{5}{n^4}\right)^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{\cancel{\sqrt{n}\cdot n^2}\left(-3+\frac{5}{n}+\frac{1}{n^2}\sin n\right)}{\cancel{n^{\frac{5}{2}}}\left(\left(1-\frac{3}{n^3}+\frac{1}{n^5}\cdot\sin n\right)^{\frac{1}{2}}+\left(1-\frac{5}{n^4}\right)^{\frac{1}{2}}\right)} \overset{\text{VOAL}}{=}$$

$$\overset{\text{VOAL}}{=} \frac{-3+0+\lim \overbrace{\frac{1}{n^2}\cdot\sin n}^{\text{„malé · omez."}}}{\lim\left(1-\frac{3}{n^3}+\underline{\frac{1}{n^5}\sin n}\right)^{\frac{1}{2}} + \lim\left(1-\frac{5}{n^4}\right)^{\frac{1}{2}}}$$

$$= \frac{-3+0+0}{(1-0+0)^{\frac{1}{2}}+(1-0)^{\frac{1}{2}}} = -\frac{3}{2}$$

(3) $$\lim_{n\to\infty} \left(\underbrace{\frac{(4n^2-4n)\cdot(2n-3)!}{(2n-1)!}}\right)^n =$$

Upravím zlomek:

$$= \frac{4n(n-1)\cdot\cancel{(2n-3)!}}{(2n-1)(2n-2)\cancel{(2n-3)!}} = \frac{2n\cancel{(2n-2)}}{(2n-1)\cancel{(2n-2)}} =$$

$$= \frac{2n-1+1}{2n-1} = 1+\frac{1}{2n-1}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{2n-1}\right)^n = \lim_{n\to\infty}\left(\left(1+\frac{1}{2n-1}\right)^{2n}\right)^{\frac{1}{2}}$$

$$= \left(\lim_{n\to\infty} \left(1+\frac{1}{2n-1}\right)^{2n-1}\left(1+\frac{1}{2n-1}\right)\right)^{\frac{1}{2}} =$$

$$\left(e\cdot\overbrace{\lim\left(1+\frac{1}{2n-1}\right)}^{=1+0}\right)^{\frac{1}{2}} = (e\cdot 1)^{\frac{1}{2}} = \sqrt{e}.$$

TEST C (30.11.2023, 8:10)

(1) $$\lim_{n\to\infty} \frac{\ln n - \frac{1}{2^n} + n!\cdot\sqrt{n}}{3n! + n^4 - \cos n}\left(\sqrt{n+5}-\sqrt{n+1}\right)$$

$$= \lim_{n\to\infty} \frac{\cancel{n!}\cdot\sqrt{n}\left(\frac{\ln n}{n!\cdot\sqrt{n}} - \frac{1}{2^n n!\sqrt{n}} + 1\right)}{\cancel{n!}\left(3 + \frac{n^4}{n!} - \frac{\cos n}{n!}\right)} \cdot$$

$$\cdot \frac{\overbrace{(n+5)-(n+1)}^{=4}}{\sqrt{n+5}+\sqrt{n+1}} \overset{\text{VOAL}}{=}$$

$$= \lim_{n\to\infty}\frac{(\dots)}{(\dots)} \cdot \lim_{n\to\infty} \frac{\sqrt{n}\cdot 4}{\sqrt{n}\left(\sqrt{1+\frac{5}{n}}+\sqrt{1+\frac{1}{n}}\right)}$$

$$\overset{\text{VOAL}}{=} \frac{0-0+1}{3+0+0}\cdot 4\cdot \frac{1}{\underbrace{\lim\sqrt{1+\frac{5}{n}}}_{=\sqrt{\lim(1+\frac{5}{n})}} + \underbrace{\lim\sqrt{1+\frac{1}{n}}}_{\text{podobně}}} =$$

$$= \frac{4}{3}\cdot\frac{1}{\sqrt{1}+\sqrt{1}} = \frac{2}{3}$$

Dílčí limity: • $\lim \frac{\ln n}{n!\cdot\sqrt{n}} = \overbrace{\lim\frac{\ln n}{\sqrt{n}}}^{\text{srov. škála} \dots = 0}\cdot\lim\frac{1}{n!}$
$= 0\cdot\frac{1}{\infty} = 0\cdot 0 = 0$

• $\lim \frac{n^4}{n!} = 0$ („škála")

• $\lim \frac{\cos n}{n!} = \lim \underbrace{\frac{1}{n!}}_{\to 0}\cdot\underbrace{\cos n}_{\text{omezená}} = 0$

(2) $$\lim_{n\to\infty} \sqrt[n]{5^{2n+1}(2+\cos n)+6^n} =: a_n$$

$$\left[\begin{array}{l} 5^{2n+1} = 5\cdot(5^2)^n = 5\cdot 25^n \\ \cos n \in [-1,1] \Rightarrow 2+\cos n \in [1,3] \end{array}\right]$$

$$25 = \sqrt[n]{25^n} \le a_n \le \sqrt[n]{5\cdot 25^n\cdot 3 + 25^n}$$

$$\sqrt[n]{16}\cdot\sqrt[n]{25^n} = 25\cdot\sqrt[n]{16}$$

2 POLICAJTI:

$$25 \Rightarrow \lim_{n\to\infty} a_n = 25 \Leftarrow 25\cdot 1$$

(3) $$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = \lim_{n\to\infty}\left(\left(\frac{n}{n-1}\right)^{-1}\right)^n =$$

$$= \lim_{n\to\infty}\left(\left(1+\frac{1}{n-1}\right)^n\right)^{-1} =$$

$$= \left(\lim_{n\to\infty}\left(1+\frac{1}{n-1}\right)^{n-1}\cdot\left(1+\frac{1}{n-1}\right)\right)^{-1}$$

$$= (e\cdot 1)^{-1} = e^{-1} = \frac{1}{e}.$$

TEST D (30. 11. 2023)

1. $$\lim_{n\to\infty}\frac{3n^{n+1}+4\log(n^3)+2n^2+13}{\sqrt[3]{n}-106n-n(n-1)^n}=$$

$$=\lim_{n\to\infty}\frac{n^{n\cancel{+1}}\left(3+\underbrace{\frac{12\log n}{n^{n+1}}}_{\to 0}+2\underbrace{\frac{n^2}{n^{n+1}}}_{\to 0}+\underbrace{\frac{13}{n^{n+1}}}_{\to 0}\right)}{\cancel{n}(n-1)^n\left(\underbrace{\frac{\sqrt[3]{n}}{n(n-1)^n}}_{\to 0}-\underbrace{\frac{106}{(n-1)^n}}_{\to 0}-1\right)}\overset{\text{VOAL}}{=}$$

$$\frac{3+0+0+0}{0-0-1}\cdot\lim_{n\to\infty}\left(\frac{n}{n-1}\right)^n=-3\lim_{n\to\infty}\left(1+\frac{1}{n-1}\right)^n$$

$$=-3\lim\left(1+\frac{1}{n-1}\right)^{n-1}\cdot\lim\left(1+\frac{1}{n-1}\right)=-3e\cdot 1$$

(Opakovaně používám růstovou škálu a VOAL)

2. $$\lim_{n\to\infty}\sqrt[n]{\underbrace{3^{2n}}_{=9^n}-n^2+10+\underbrace{5^{n+4}}_{5^4\cdot 5^n}+4\Big(\underbrace{\frac{80}{9}}_{80/9\,<\,9}\Big)^n}\cdot\left(\sqrt{n+1}-\sqrt{n+2}\right)\sqrt{n}=$$

$$=\lim_{n\to\infty}\sqrt[n]{9^n\left(1-\frac{n^2}{9^n}+\frac{10}{9^n}+\frac{5^{n+4}}{9^n}+4\cdot\left(\frac{80}{81}\right)^n\right)}\cdot\frac{(n+1)-(n+2)}{\sqrt{n+1}+\sqrt{n+2}}\cdot\sqrt{n}$$

$$=\lim_{n\to\infty}9\cdot\frac{-\sqrt{n}}{\sqrt{n}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}\right)}\cdot\sqrt[n]{1-\frac{n^2}{9^n}+\frac{10}{9^n}+\frac{5^{n+4}}{9^n}+4\left(\frac{80}{81}\right)^n}$$

$$\overset{\text{VOAL}}{=}-9\cdot\frac{1}{\underbrace{\lim\sqrt{1+\frac{1}{n}}}_{=\sqrt{\lim(1+\frac{1}{n})}=\sqrt{1+0}}+\lim\sqrt{1+\frac{2}{n}}}\cdot\lim\sqrt[n]{1-\dots}$$

$$=\frac{-9}{2}\cdot\lim\sqrt[n]{1-\dots}=-\frac{9}{2}$$

2 POLICAJTI:

$$\sqrt[n]{\frac{1}{2}}\le\sqrt[n]{1\underbrace{-\frac{n^2}{9^n}}_{<0\,!}+\underbrace{\frac{10}{9^n}}_{\to 0}+\underbrace{\frac{5^{n+4}}{9^n}}_{\to 0}+\underbrace{4\left(\frac{80}{81}\right)^n}_{\to 0}}\le\sqrt[n]{4}$$

[od n_0 dál] $n\ge n_0$

Členy jdoucí k 0 jsou od jistého indexu n_0 všechny menší než 1.

Ale $\lim\limits_{n\to\infty}\sqrt[n]{4}=1$, $\lim\limits_{n\to\infty}\sqrt[n]{\frac{1}{2}}=\frac{1}{\lim\sqrt[n]{2}}=1$

Celkem: $\lim\sqrt[n]{\dots}=1$.

3. $$\lim_{x\to 1}\frac{2x^3-x^2-4x+3}{x^3-x^2-x+1}=$$

$(2x^3-x^2-4x+3):(x-1)=2x^2+x-3$
$-(2x^3-2x^2)$
$x^2-4x+3=(x-1)\cdot(x-3)$

$(2x^2+x-3)=(x-1)(2x+3)$

Tedy $2x^3-x^2-4x+3=(x-1)^2(2x+3)$

$(x^3-x^2-x+1):(x-1)=x^2-1=(x-1)(x+1)$
$-(x^3-x^2)$
$-x+1$

Tedy $x^3-x^2-x+1=(x-1)^2(x+1)$.

$$=\lim_{x\to 1}\frac{(x-1)^2(2x+3)}{(x-1)^2(x+1)}=\lim_{x\to 1}\frac{2x+3}{x+1}=\frac{2\cdot 1+3}{1+1}$$

nedělíme 0 ⇒ spojitá v 1

$$=\frac{5}{2}$$

TEST E (1.12. 2023, 10:40)

1. $\lim\limits_{n\to\infty}\left(1+\frac{1}{3n}\right)^n\left(\sqrt[3]{2n^3-n^2}-\sqrt[3]{2n^3+2n^2}\right)$

Rozdělíme na 2 části:

- $\lim\limits_{n\to\infty}\left(1+\frac{1}{3n}\right)^n = \lim \sqrt[3]{\left(1+\frac{1}{3n}\right)^{3n}} = \sqrt[3]{\lim\left(1+\frac{1}{3n}\right)^{3n}} = \sqrt[3]{e}$

- $\lim\limits_{n\to\infty}\left(\sqrt[3]{2n^3-n^2}-\sqrt[3]{2n^3+2n^2}\right)$

$$= \lim_{n\to\infty}\frac{2n^3-n^2-(2n^3+2n^2)}{(2n^3-n^2)^{2/3}+(2n^3-n^2)^{1/3}(2n^3+2n^2)^{1/3}+(2n^3+2n^2)^{2/3}}$$

$$= \lim_{n\to\infty}\frac{-3n^2}{n^2\left(2-\frac{1}{n}\right)^{2/3}+n\left(2-\frac{1}{n}\right)^{1/3}\cdot n\left(2+\frac{2}{n}\right)^{1/3}+n^2\left(2+\frac{2}{n}\right)^{2/3}}$$

$$= \lim_{n\to\infty}\frac{-3\cancel{n^2}}{\cancel{n^2}\left(\left(2-\frac{1}{n}\right)^{2/3}+\left(2-\frac{1}{n}\right)^{1/3}\left(2+\frac{2}{n}\right)^{1/3}+\left(2+\frac{2}{n}\right)^{2/3}\right)}$$

$$\overset{\text{VOAL}}{=}\frac{-3}{2^{2/3}+2^{1/3}\cdot 2^{1/3}+2^{2/3}} = \frac{-3}{3\cdot 2^{2/3}} = -2^{-2/3}$$

Dílčí výpočty: (např.)

- $(2n^3-n^2)^{2/3} = \left(n^3\left(2-\frac{1}{n}\right)\right)^{2/3} = (n^3)^{2/3}\cdot\left(2-\frac{1}{n}\right)^{2/3} = n^2\left(2-\frac{1}{n}\right)^{2/3}$. APOD.
- $\lim\limits_{n\to\infty}\left(2-\frac{1}{n}\right)^{2/3} = \left(\lim\limits_{n\to\infty}\left(2-\frac{1}{n}\right)\right)^{2/3} = (2-0)^{2/3} = 2^{2/3}$ ...

Celkem: $\lim\cdots = \sqrt[3]{e}\cdot\left(-2^{-2/3}\right) = -\frac{\sqrt[3]{e}}{\sqrt[3]{4}}$

2. $\lim\limits_{n\to\infty}\frac{5n^2}{6n^3+2}\cdot\sqrt[n]{(n+1)^n+n!+10^{2n}} =$

$$= \lim_{n\to\infty}\frac{5n^2}{n^3\left(6+\frac{2}{n^3}\right)}\cdot(n+1)\cdot\sqrt[n]{1+\frac{n!}{(n+1)^n}+\frac{100^n}{(n+1)^n}}$$

$$= 5\lim_{n\to\infty}\frac{\cancel{n^3}\left(1+\frac{1}{n}\right)}{\cancel{n^3}\left(6+\frac{2}{n^3}\right)}\cdot\sqrt[n]{\cdots}\overset{\text{VOAL}}{=}5\cdot\frac{1+0}{6+0}\cdot\lim\sqrt[n]{\cdots}$$

Lemma o 2 policajtech:

$$1=\sqrt[n]{1}\le\sqrt[n]{1+\frac{n!}{(n+1)^n}+\frac{100^n}{(n+1)^n}}\overset{\text{od jistého } n_0}{\le}\sqrt[n]{3}\to 1$$

Protože $n \ll n^n < (n+1)^n$,
$100^n \ll n^n < (n+1)^n$, existuje $n_0\in\mathbb{N}$,
že $\forall n\ge n_0:\ \frac{n!}{(n+1)^n}<1\ \wedge\ \frac{100^n}{(n+1)^n}<1$.

Tedy (podle Lo2P):
$\lim\limits_{n\to\infty}\sqrt[n]{\cdots}=1$.

Celkem: $\lim\cdots = \frac{5}{6}\cdot 1 = \frac{5}{6}$.

3. $\lim\limits_{n\to\infty}\frac{\sqrt[4]{n^6+4n^3}-\sqrt[6]{2n^9-n^4}}{\sqrt{3n^3-4n}-\sqrt[4]{n^4+1}} =$

$$= \lim_{n\to\infty}\frac{n^{\frac{6}{4}}\cdot\sqrt[4]{1+\frac{4}{n^3}}-n^{\frac{9}{6}}\sqrt[6]{2-\frac{1}{n^5}}}{n^{\frac{3}{2}}\cdot\sqrt{3-\frac{4}{n^2}}-n\cdot\sqrt[4]{1+\frac{1}{n^4}}} =$$

$$= \lim_{n\to\infty}\frac{\cancel{n^{\frac{6}{4}}}\left(\sqrt[4]{1+\frac{4}{n^3}}-\sqrt[6]{2-\frac{1}{n^5}}\right)}{\cancel{n^{\frac{3}{2}}}\left(\sqrt{3-\frac{4}{n^2}}-\frac{1}{\sqrt{n}}\cdot\sqrt[4]{1+\frac{1}{n^4}}\right)} =$$

$$\overset{\text{VOAL}}{=}\frac{\sqrt[4]{1+0}-\sqrt[6]{2-0}}{\sqrt{3-0}-\underbrace{\frac{1}{\infty}\cdot\sqrt[4]{1+0}}_{=0\cdot 1=0}} = \frac{1-\sqrt[6]{2}}{\sqrt{3}}$$

Dílčí limity po aplikaci VOAL: (např.)

- $\lim\limits_{n\to\infty}\sqrt[4]{1+\frac{4}{n^3}} = \sqrt[4]{\lim\limits_{n\to\infty}\left(1+\frac{4}{n^3}\right)} =$

$= \sqrt[4]{1+\frac{4}{\infty^3}} = \sqrt[4]{1+\frac{4}{\infty}} = \sqrt[4]{1+0} = 1$,

a podobně v ostatních případech.

TEST F (1.12.2023)

1. $$\lim_{n\to\infty}\left(\sqrt[3]{n^3+2n+1}-\sqrt[3]{n^3+2n}\right)\cdot\frac{\sin n+n^3+8n^5-\sqrt{n}}{\log(5^n)+2n^3+\sqrt[4]{n^5+2}}$$

$[\log(5^n)=n\cdot\log 5]$

$$=\lim_{n\to\infty}\frac{(n^3+2n+1)-(n^3+2n)}{(n^3+2n+1)^{2/3}+(n^3+2n+1)^{1/3}(n^3+2n)^{1/3}+(n^3+2n)^{2/3}}\cdot\frac{n^5\left(\frac{1}{n^5}\sin n+\frac{1}{n^2}+8-\frac{\sqrt{n}}{n^5}\right)}{n^3\cdot\left(\frac{\log 5}{n^2}+2+\frac{n^{5/4}}{n^3}\cdot\sqrt[4]{1+\frac{2}{n^5}}\right)}=$$

$$=\lim_{n\to\infty}\frac{1}{n^2\left(\left(1+\frac{2}{n^2}+\frac{1}{n^3}\right)^{2/3}+\left(1+\frac{2}{n^2}+\frac{1}{n^3}\right)^{1/3}\left(1+\frac{2}{n^2}\right)^{1/3}+\left(1+\frac{2}{n^2}\right)^{2/3}\right)}\cdot n^2\cdot\lim\frac{(\dots+8-\dots)}{(\dots+2+\dots)}=$$

$$=\lim\frac{n^2}{n^2}\cdot\lim\frac{1}{(\ )+(\ )(\ )+(\ )}\cdot\lim\frac{(\ )}{(\ )}=$$

$$=1\cdot\frac{1}{1^{2/3}+1^{1/3}1^{1/3}+1^{2/3}}\cdot\frac{0+0+8-0}{0+2+0\cdot\sqrt[4]{1+0}}=\infty\cdot\frac{1}{3}\cdot 4=\frac{4}{3}$$

2. $$\lim_{n\to\infty}\left(\frac{(n+2)!}{n!\,(n^2+2n)}\right)^{4n+1}=$$

$$=\lim_{n\to\infty}\left(\frac{(n+2)(n+1)}{n^2+2n}\right)^{4n+1}=\lim_{n\to\infty}\left(\frac{n^2+3n+2}{n^2+2n}\right)^{4n+1}$$

$$=\lim_{n\to\infty}\left(1+\frac{\cancel{n+2}}{n\cancel{(n+2)}}\right)^{4n+1}=\lim\left(1+\frac{1}{n}\right)^{4n}\cdot\left(1+\frac{1}{n}\right)$$

$$=\lim\left(1+\frac{1}{n}\right)\cdot\left(\lim\left(1+\frac{1}{n}\right)\right)^4=1\cdot e^4=e^4.$$

3. $$\lim_{x\to 4}\frac{\sqrt{5x-4}-4}{x^2-2x-8}\cdot\frac{2x^3+3x-50}{x^3-5x-40}=$$

VOAL+

dosazení do spojité

- $x^2-2x-8=(x-4)(x+2)$
- $2\cdot 4^3+3\cdot 4-50=2\cdot 64+12-50=140-50=90$
- $4^3-5\cdot 4-40=64-20-40=4$
- $\sqrt{5x-4}-4=\frac{5x-4-16}{\sqrt{5x-4}+4}=\frac{5x-20}{\sqrt{5x-4}+4}$

$$=\frac{90}{4}\cdot\lim_{x\to 4}\frac{5\cancel{(x-4)}}{\cancel{(x-4)}(x+2)\left(\sqrt{5x-4}+4\right)}=$$

$$=\frac{450}{4}\cdot\frac{1}{(4+2)\cdot\left(\sqrt{16}+4\right)}=\frac{450}{4\cdot 48}=\frac{150}{64}=\frac{75}{32}$$