1.

$$\lim_{n\to\infty} \frac{\left(\ln(n^2)\right)^2}{\sqrt{n+5\ln^2 n}-\sqrt{n+2\ln^2 n}}\cdot\frac{1}{\sqrt{n}} =$$

$$= \lim_{n\to\infty} \frac{(2\ln n)^2}{(n+5\ln^2 n)-(n+2\ln^2 n)}\cdot\frac{\sqrt{n+5\ln^2 n}+\sqrt{n+2\ln^2 n}}{\sqrt{n}} =$$

$$= \lim_{n\to\infty} \frac{4\cdot\cancel{\ln^2 n}}{3\cdot\cancel{\ln^2 n}}\cdot\frac{\cancel{\sqrt{n}}\left(\sqrt{1+\frac{5\ln^2 n}{n}}+\sqrt{1+\frac{2\ln^2 n}{n}}\right)}{\cancel{\sqrt{n}}} =$$

$$= \frac{4}{3}\lim_{n\to\infty}\left(\sqrt{1+\frac{5\ln^2 n}{n}}+\sqrt{1+\frac{2\ln^2 n}{n}}\right) = \frac{4}{3}\left(\sqrt{1+0}+\sqrt{1+0}\right) = \underline{\underline{\frac{8}{3}}}$$

$$\left[\ \lim \frac{\ln^2 n}{n} = \lim \frac{\ln n}{\sqrt{n}}\cdot\frac{\ln n}{\sqrt{n}} = 0\cdot 0 = 0.\ \right]$$

2.

$$\lim_{x\to 0_+} \frac{\cos\sqrt{x}-1}{e^x-1}\cdot \underbrace{\operatorname{arctg}\underbrace{\frac{1}{x^2}}_{\to\infty}}_{\frac{\pi}{2}}\cdot\underbrace{\ln x}_{\to-\infty}\cdot\operatorname{arctg} x \overset{\text{VOLSF}}{=}$$

$$= \frac{\pi}{2}\lim_{x\to 0_+}\frac{\cos\sqrt{x}-1}{x}\cdot\underbrace{\frac{x}{e^x-1}}_{\to\frac{1}{1}=1}\cdot\underbrace{\frac{\operatorname{arctg} x}{x}}_{\to 1}\cdot x\cdot\ln x \overset{\text{VOLSF}}{=}$$

$$= \frac{\pi}{2}\cdot\frac{-1}{2}\cdot 1\cdot 1\cdot\lim_{x\to 0_+} x\cdot\ln x = \frac{-\pi}{4}\lim_{x\to 0_+}\frac{\ln x}{\frac{1}{x}} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{l'H}{=}}$$

$$= -\frac{\pi}{4}\lim_{x\to 0_+}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = -\frac{\pi}{4}\lim_{x\to 0_+}(-x) = -\frac{\pi}{4}\cdot 0 = \underline{\underline{0}}$$

VOLSF1: $g(x)=\frac{1}{x^2}\xrightarrow{x\to 0_+}\infty$, $f(y)=\operatorname{arctg} y\xrightarrow{y\to\infty}\frac{\pi}{2}$

(P): $g(x)\neq\infty$ $\forall x\in(0,\infty)$ ... stačí.

VOLSF2: $g(x)=\sqrt{x}\xrightarrow{x\to 0_+}0$, $f(y)=\frac{\cos y-1}{y^2}=-\frac{1-\cos y}{y^2}\xrightarrow{y\to 0_+}-\frac{1}{2}$

(P): $\sqrt{x}\neq 0$ pro $x\in(0,\infty$

3.
$$\sum_{n=1}^{\infty} \frac{n^n (n+1)^4}{3^n \cdot n! \cdot n^6} =: \sum_{n=1}^{\infty} a_n$$

Limitní podílové kritérium:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^{n+1} (n+2)^4}{3^{n+1} \cdot (n+1)! \cdot (n+1)^6} \cdot \frac{3^n \cdot n! \cdot n^6}{n^n (n+1)^4} =$$

$$= \lim_{n\to\infty} \frac{(n+1)^n \cdot (n+1)}{n^n \cdot 3} \cdot \underbrace{\frac{(n+2)^4}{(n+1)^4}}_{\to 1} \cdot \underbrace{\frac{n^6}{(n+1)^6}}_{\to 1} \cdot \frac{n!}{(n+1)!} =$$

$$= \left(\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n\right) \cdot \frac{1}{3} \cdot 1 \cdot 1 \cdot \lim \frac{(n+1)!}{(n+1)!} =$$

$$= \frac{1}{3} \lim \left(1 + \frac{1}{n}\right)^n = \frac{e}{3} < 1 \implies$$

$\implies$ podle d'Alembertova kritéria (podílového) $\sum_{n=1}^{\infty} a_n$ K.

4. $f(x) = \dfrac{x}{\ln x}$

2 $D_f = (0,\infty)\setminus\{1\} = \cancel{(0,\infty)}\ (0,1)\cup(1,\infty)$

spojitá na D_f, symetrie nejsou.

$\lim\limits_{x\to 0_+} f(x) = \dfrac{0}{-\infty} = 0$, $\lim\limits_{x\to 1_-} f(x)\ ``=" \dfrac{1}{0_-}" = -\infty$,

3

$\lim\limits_{x\to\infty} f(x) \overset{l'H}{\underset{\frac{\infty}{\infty}}{=}} \lim\limits_{x\to\infty} \dfrac{1}{\frac{1}{x}} = \infty$, $\lim\limits_{x\to 1_+} f(x)\ ``=" \dfrac{1}{0_+}" = +\infty$.

Asymptota v ∞: $a = \lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{x}{x\cdot\ln x} = 0$

1

$b = \lim\limits_{x\to\infty} (f(x) - 0\cdot x) = \lim\limits_{x\to\infty} f(x) = \infty \Rightarrow$

$\Rightarrow$ asymptota není.

2 $f'(x) = \dfrac{1\cdot\ln x - x\cdot\frac{1}{x}}{\ln^2 x} = \dfrac{\ln x - 1}{\underbrace{\ln^2 x}_{>0\ (x\in D_f)}} > 0 \Leftrightarrow \ln x > 1$

$\Leftrightarrow x > e$.

2

	$(0,1)$	$(1,e)$	(e,∞)
f'	$-$	$-$	$+$
f	↘	↘	↗

V bodě $x = e$ je lokální minimum,

$f(e) = \dfrac{e}{\ln e} = e$

$f''(x) = \dfrac{\frac{1}{x}\ln^2 x - (\ln x - 1)\,2\ln x\cdot\frac{1}{x}}{(\ln^2 x)^2} = \dfrac{\ln^2 x - 2\ln^2 x + 2\ln x}{x(\ln^2 x)^2} =$

2

$= \dfrac{2\ln x - \ln^2 x}{x\ln^4 x} > 0 \Leftrightarrow 2\ln x - \ln^2 x > 0$

$(x\in D_f)$

$\Leftrightarrow \quad \ln x \cdot (2 - \ln x) > 0$

$\Leftrightarrow \left(\ln x > 0 \ \wedge \ \ln x < 2\right) \vee \left(\ln x < 0 \ \wedge \ \ln x > 2\right)$

$\Leftrightarrow \quad x \in (1, e^2)$

3

	$(0,1)$	$(1,e^2)$	(e^2,∞)
f''	$\ominus$	$\oplus$	$\ominus$
f	$\cap$	$\cup$	$\cap$

e^2 je inflexní bod

$f(e^2) = \dfrac{e^2}{\ln e^2} = \dfrac{e^2}{2}$

5