A1) $$\lim_{n\to\infty} \frac{(n+1)^n + n! + n^5}{n^{n-1} + 5^n}\left(\sqrt{n^2+1} - n\right) =$$

$$= \lim_{n\to\infty} \frac{(n+1)^n\left(1 + \frac{n!}{(n+1)^n} + \frac{n^5}{(n+1)^n}\right)}{n^{n-1}\left(1 + 5\cdot\frac{5^{n-1}}{n^{n-1}}\right)} \cdot \frac{1}{\sqrt{n^2+1} + n} =$$

$$= \lim_{n\to\infty} \frac{1 + \frac{n!}{(n+1)^n} + \frac{n^5}{(n+1)^n}}{1 + 5\cdot\frac{5^{n-1}}{n^{n-1}}} \cdot \underbrace{\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n}_{=e} \cdot \lim \frac{n}{n\left(\sqrt{1+\frac{1}{n^2}} + 1\right)} =$$

$$= \frac{1+0+0}{1+5\cdot 0} \cdot e \cdot \frac{1}{\sqrt{1+0} + 1} = 1\cdot e\cdot\frac{1}{2} = \frac{e}{2}$$

A2) $$\lim_{n\to\infty} \underbrace{\sqrt[n]{3^{2n} + 5^n + 7^{n+2} + n^n}\cdot\frac{1}{n^2}}_{\text{označme } a_n.\ \text{Pak:}}$$

$$\left[\begin{array}{l} 3^{2n} = 9^n \le n^n \\ 5^n \le n^n \\ 7^{n+2} = 49\cdot 7^n \le n^n \\ \text{od jistého } n_0 \end{array}\right]$$

$$\sqrt[n]{n^n}\cdot\frac{1}{n^2} \le a_n \le \sqrt[n]{n^n + n^n + 49n^n + n^n}\cdot\frac{1}{n^2}$$

$$\sqrt[n]{n^n}\cdot\frac{1}{n^2} = n\cdot\frac{1}{n^2} = \frac{1}{n} \to 0$$

$$\sqrt[n]{n^n + n^n + 49n^n + n^n}\cdot\frac{1}{n^2} = \sqrt[n]{52}\cdot\sqrt[n]{n^n}\cdot\frac{1}{n^2} = \sqrt[n]{52}\cdot\frac{1}{n} \to 1\cdot 0$$

$$\Rightarrow \quad a_n \xrightarrow{(\text{LO2P})} 0 \quad \Leftarrow$$

A3)

$$\lim_{m\to\infty}\frac{\sqrt[4]{m^5+1}-\sqrt[3]{m^4+m^2}}{\sqrt[6]{8m^8+100m^7}-\sqrt[5]{3m^6+5m^4}}=\left[\frac{\frac{4}{3}>\frac{5}{4}}{\frac{8}{6}=\frac{4}{3}>\frac{6}{5}}\right]$$

$$=\lim_{m\to\infty}\frac{\sqrt[4]{1+\frac{1}{m^5}}\cdot m^{\frac{5}{4}}-\sqrt[3]{1+\frac{1}{m^2}}\cdot m^{\frac{4}{3}}}{\sqrt[6]{8+\frac{100}{m}}\cdot m^{\frac{8}{6}}-\sqrt[5]{3+\frac{5}{m^2}}\cdot m^{\frac{6}{5}}}=\left[\begin{array}{l}\frac{5}{4}-\frac{4}{3}=\frac{15-16}{12}\\ \frac{6}{5}-\frac{4}{3}=\frac{18-20}{15}\end{array}\right]$$

$$=\lim_{m\to\infty}\frac{m^{\frac{4}{3}}\left(-\sqrt[3]{1+\frac{1}{m^2}}+\sqrt[4]{1+\frac{1}{m^5}}\cdot x^{-\frac{1}{12}}\right)}{m^{\frac{4}{3}}\left(\sqrt[6]{8+\frac{100}{m}}-\sqrt[5]{3+\frac{5}{m^2}}\cdot x^{-\frac{2}{15}}\right)}=$$

$$\frac{-\sqrt[3]{1+0}+\sqrt[4]{1+0}\cdot 0}{\sqrt[6]{8+0}-\sqrt[5]{3+0}\cdot 0}=\frac{-1}{\sqrt[6]{8}}=\frac{-1}{\sqrt[6]{2^3}}=-2^{-\frac{3}{6}}=-2^{-\frac{1}{2}}=\underline{\underline{-\frac{\sqrt{2}}{2}}}$$

B1)

$$\lim_{m\to\infty}\left(\sqrt[3]{m^4+2m^3+m+6}-\sqrt[3]{m^4+2m^3-m}\right)=\left[\begin{array}{l}a-b=\\ =a-b\cdot\frac{a^2+ab+b^2}{a^2+ab+b^2}=\\ =\frac{a^3-b^3}{a^2+ab+b^2}\end{array}\right]$$

$$=\lim_{m\to\infty}\frac{m^4+2m^3+m+6-(m^4+2m^3-m)}{(m^4+2m^3+m+6)^{\frac{2}{3}}+(m^4+2m^3+m+6)^{\frac{1}{3}}(m^4+2m^3-m)^{\frac{1}{3}}+(m^4+2m^3-m)^{\frac{2}{3}}}$$

$$=\lim_{m\to\infty}\frac{2m+6}{m^{8/3}\left(1+\frac{2}{m}+\frac{1}{m^3}+\frac{6}{m^4}\right)^{\frac{2}{3}}+m^{\frac{4}{3}}\left(1+\frac{2}{m}+\ldots\right)^{\frac{1}{3}}m^{\frac{4}{3}}\left(1+\frac{2}{m}+\ldots\right)^{\frac{1}{3}}+m^{\frac{8}{3}}\left(1+\frac{2}{m}-\ldots\right)^{\frac{2}{3}}}$$

$$=\lim\frac{2m+6}{m^{8/3}}\cdot\frac{1}{(\ )^{2/3}+(\ )^{1/3}\cdot(\ )^{1/3}+(\ )^{2/3}}=\left(\lim\frac{2+\frac{6}{m}}{m^{5/3}}\right)\cdot\frac{1}{1+1+1}=$$

$$=\frac{2+0}{\infty}\cdot\frac{1}{3}=0$$

B2) $$\lim_{n\to\infty} \frac{(n+1)! \cdot (1+2+3+\ldots+n)}{(n^2+5+n!+\ln n)\,n^k}, \quad k \in \mathbb{N}$$

$$= \lim_{n\to\infty} \frac{(n+1)!\ \frac{n(n+1)}{2}}{n!\left(\frac{n^2}{n!}+\frac{5}{n!}+1+\frac{\ln n}{n!}\right)n^k} =$$

$$= \lim_{n\to\infty} \frac{1}{\frac{n^2}{n!}+\frac{5}{n!}+1+\frac{\ln n}{n!}} \cdot \frac{1}{2}\lim_{n\to\infty} \frac{(n+1)!\cdot n(n+1)}{n!\cdot n^k} =$$

$$= \frac{1}{0+0+1+0} \cdot \frac{1}{2} \cdot \lim \frac{(n+1)^2}{n^{k-1}} = \frac{1}{2} \lim \frac{n^2\left(1+\frac{1}{n}\right)^2}{n^{k-1}} =$$

$$= \frac{1}{2}\cdot(1+0)^2 \cdot \lim \frac{n^2}{n^{k-1}} = \frac{1}{2} \lim n^{3-k} =$$

$$= \begin{cases} \frac{1}{2}\cdot 0 = 0, & \text{pokud } k>3; \\ \frac{1}{2}\cdot 1 = \frac{1}{2}, & \text{pokud } k=3; \\ \frac{1}{2}\cdot\infty = \infty, & \text{pokud } k<3. \end{cases}$$

B3) $$\lim_{x\to 1} \frac{2x^3+3x^2-3x-2}{x^3+x-x^2-1} = \left[\begin{array}{l}\text{dosazením } x=1 \rightsquigarrow \frac{0}{0}. \\ \text{Tedy 1 je kořen obou polynomů}\end{array}\right]$$

$$(2x^3+3x^2-3x-2):(x-1) = 2x^2+5x+2$$
$$\begin{array}{l} -(2x^3-2x^2) \\ \hline \quad 5x^2-3x-2 \\ \quad -(5x^2-5x) \\ \hline \qquad 2x-2 \end{array}$$

$$x^3+x-x^2-1 = x^3-x^2+x-1 = x^2(x-1)+(x-1) = (x-1)(x^2+1).$$

$$= \lim_{x\to 1} \frac{(x-1)(2x^2+5x+2)}{(x-1)(x^2+1)} = \lim_{x\to 1} \frac{2x^2+5x+2}{x^2+1} = \frac{2\cdot 1^2+5\cdot 1+2}{1^2+1} = \underline{\underline{\frac{9}{2}}}$$

1. Zápočtový test A (VZOR)

ZS 2021/2022

Spočtěte následující limity.

(1) $$\lim_{n\to\infty} \frac{(n+1)^n + n! + n^5}{n^{n-1} + 5^n}\left(\sqrt{n^2+1} - n\right).$$

(2) $$\lim_{n\to\infty} \sqrt[n]{3^{2n} + 5^n + 7^{n+2} + n^n} \cdot \frac{1}{n^2}$$

(3) $$\lim_{n\to\infty} \frac{\sqrt[4]{n^5+1} - \sqrt[3]{n^4+n^2}}{\sqrt[6]{8n^8 + 100n^7} - \sqrt[5]{3n^6+5n^4}}$$

1. Zápočtový test B (VZOR)

ZS 2021/2022

Spočtěte následující limity.

(1) $$\lim_{n\to\infty} \left(\sqrt[3]{n^4 + 2n^3 + n + 6} - \sqrt[3]{n^4 + 2n^3 - n}\right)$$

(2) $$\lim_{n\to\infty} \frac{(n+1)! \cdot (1 + 2 + 3 + \ldots + n)}{(n^2 + 5 + n! + \ln n)\, n^k}; \quad k \in \mathbb{N} \text{ je parametr}$$

(3) $$\lim_{x\to 1} \frac{2x^3 + 3x^2 - 3x - 2}{x^3 + x - x^2 - 1}$$