① $$\lim_{x\to 0} \frac{\cos x - \sqrt{1-x^2} - \frac{x^4}{6}}{x^5} = \lim_{x\to 0} \frac{o(x^5)}{x^5} = \underline{\underline{0}}\,.$$

T.P. 5. řádu čitatele:

$$(1+y)^{\frac{1}{2}} = 1 + \binom{\frac{1}{2}}{1} y + \binom{\frac{1}{2}}{2} y^2 + \binom{\frac{1}{2}}{3} y^3 + o(y^3),\ y \to$$

$$\binom{\frac{1}{2}}{1} = \frac{1}{2},\quad \binom{\frac{1}{2}}{2} = \frac{\frac{1}{2}\cdot\left(-\frac{1}{2}\right)}{2!} = -\frac{1}{8},$$

$$\binom{\frac{1}{2}}{3} = \binom{\frac{1}{2}}{2}\cdot\frac{-\frac{3}{2}}{3} = \frac{1}{16}.$$

Tedy:

$$\sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 + o(x^6),\ x\to 0 =$$

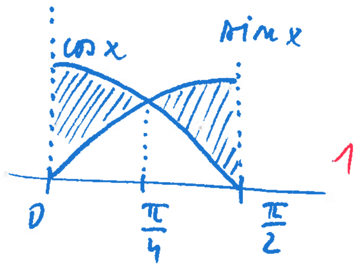
$$= 1 - \frac{x^2}{2} - \frac{x^4}{8} + o(x^5),\ x\to 0.$$

$$\underline{\text{ČIT.}} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \left(1 - \frac{x^2}{2} - \frac{x^4}{8}\right) - \frac{x^4}{6} + o(x^5),\ x\to 0 =$$

$$= \frac{x^4}{24} + \frac{3x^4}{24} - \frac{4x^4}{24} + o(x^5),\ x\to 0 = o(x^5),\ x \to 0.$$

③ $$V = 2\cdot\left(\int_0^{\frac{\pi}{4}} \pi \cos^2 x\,dx - \int_0^{\frac{\pi}{4}} \pi \sin^2 x\,dx\right)$$

3



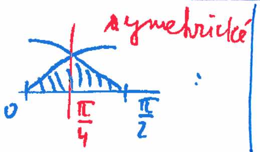
1

$$= 2\pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)\,dx = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x\,dx =$$

$$= 2\pi \left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} = \pi\left(\sin\frac{\pi}{2} - \sin 0\right) = \pi$$

(Tedy $V = \pi$ j³.)

2. VARIANTA



$$V = 2\pi \int_0^{\frac{\pi}{4}} \sin^2 x\,dx =$$

$\cos 2x =$
$\cos^2 x - \sin^2 x =$
$1 - 2\sin^2 x$
$\Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$

$$= 2\pi \int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{2}\,dx = \cancel{2}\pi\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} =$$

$$= \pi\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2} - (0-0)\right) = \frac{\pi^2}{4} - \frac{\pi}{2} = \frac{\pi}{2}\left(\frac{\pi}{2} - 1\right)$$

② $$\int \frac{\sqrt{x}-\sqrt{x-2}}{\sqrt{x}+\sqrt{x-2}}\,dx = \int \frac{1-\sqrt{\frac{x-2}{x}}}{1+\sqrt{\frac{x-2}{x}}}\,dx \mapsto [(*)] =$$

$y=\sqrt{\frac{x-2}{x}}$

$y^2\cdot x = x-2$

$x(y^2-1) = -2$

$x = \frac{2}{1-y^2}$

$dx = \frac{4y}{(1-y^2)^2}\,dy$ (*)

$$= \int \frac{1-y}{1+y}\cdot\frac{4y}{(1-y^2)^2}\,dy =$$

$$= \int \frac{4y}{(1+y)^3(1-y)}\,dy = \frac{1}{2}\int\frac{dy}{1+y} + \int\frac{dy}{(1+y)^2} - 2\int\frac{dy}{(1+y)^3} + \frac{1}{2}\int\frac{dy}{1-y}$$

PARC. ZLOMKY: $\frac{4y}{(1+y)^3(1-y)} = \frac{A}{1+y} + \frac{B}{(1+y)^2} + \frac{C}{(1+y)^3} + \frac{D}{1-y} \Rightarrow$

$$4y = A(1+y)^2(1-y) + B(1+y)(1-y) + C(1-y) + D(1+y)^3$$

$y=1$: $4 = 0 + 0 + 0 + 8D \Rightarrow D = \frac{1}{2}$

$y=-1$: $-4 = 2C \Rightarrow C = -2$

$y=0$: $0 = A + B - 2 + \frac{1}{2}$

$y=2$: $8 = -9A - 3B + 2 + \frac{27}{2}$

$A + B = \frac{3}{2}$

$9A + 3B = \frac{15}{2}$

$6A = \frac{6}{2}$

$A = \frac{1}{2}$

$B = 1$

$$\stackrel{c}{=} \frac{1}{2}\ln|1+y| - \frac{1}{1+y} + \frac{1}{(1+y)^2} - \frac{1}{2}\ln|1-y|$$

$$= \frac{1}{2}\ln\left|\frac{1+\sqrt{\frac{x-2}{x}}}{1-\sqrt{\frac{x-2}{4}}}\right| - \frac{1}{1+\sqrt{\frac{x-2}{x}}} + \frac{1}{\left(1+\sqrt{\frac{x-2}{x}}\right)^2}$$