1. Jinak: $\int \frac{dx}{\sin x \cdot \cos x} = 2\int \frac{dx}{\sin 2x} = \left| \begin{array}{l} t = 2x \\ dt = 2dx \end{array} \right| = \int \frac{dt}{\sin t} =$

$$= \int \frac{\sin t \, dt}{\sin^2 t} = -\int \frac{-\sin t \, dt}{(1-\cos^2 t)} = \left| \begin{array}{l} z = \cos t \\ dz = -\sin t \, dt \end{array} \right| = -\int \frac{dz}{1-z^2}$$

$$= -\int \frac{dz}{(1-z)(1+z)} = \int \frac{dz}{(z-1)(z+1)} = \frac{1}{2}\int \frac{dz}{z-1} - \frac{1}{2}\int \frac{dz}{z+1} \stackrel{c}{=}$$

$$\frac{A}{z-1} + \frac{B}{z+1} = \frac{1}{\cdots}\left(Az + A + Bz - B\right)$$

$$\Rightarrow A+B = 0 \qquad A - B = 1$$

$$\underline{A = \tfrac{1}{2}}, \ \underline{B = -\tfrac{1}{2}}.$$

$$\stackrel{c}{=} \frac{1}{2}\ln\left|\frac{z-1}{z+1}\right| = \ln\left|\frac{\cos 2x - 1}{\cos 2x + 1}\right|^{\frac{1}{2}} = \ln\left|\frac{\cos^2 x - \sin^2 x - 1}{\cos^2 x - \sin^2 x + 1}\right|^{\frac{1}{2}} =$$

$$= \ln\left|\frac{-\sin^2 x - \sin^2 x}{2\cos^2 x}\right|^{\frac{1}{2}} = \ln\left|-\operatorname{tg}^2 x\right|^{\frac{1}{2}} = \ln\left|\operatorname{tg} x\right|$$

$$\left(x \notin \left\{\tfrac{k\pi}{2} : k \in \mathbb{Z}\right\}\right)$$

1. nejjednodušeji: $\int \frac{dx}{\sin x \cdot \cos x} = \int \frac{\cos x}{\sin x} \cdot \frac{dx}{\cos^2 x} = \left| \begin{array}{l} t = \operatorname{tg} x \\ dt = \frac{1}{\cos^2 x} dx \end{array} \right|$

$$= \int \frac{dt}{t} \stackrel{c}{=} \ln|t| = \ln|\operatorname{tg} x|.$$

① $y' \sin x \cdot \cos x - y = 0$ $\qquad \dots \qquad y \equiv 0$ S.Ř. na $\mathbb{R}$

$$y' = \frac{y}{\sin x \cos x} \qquad x \notin \left\{ \frac{k\pi}{2} : k \in \mathbb{Z} \right\}$$

$$\frac{y'}{y} = \frac{1}{\sin x \cdot \cos x} \qquad y \neq 0$$

$$\ln|y| \overset{c}{=} \int \frac{dx}{\sin x \cos x} = \int \frac{\cos x \, dx}{\sin x\,(1-\sin^2 x)} = \left| \begin{array}{l} t = \sin x \\ dt = \cos x \, dx \end{array} \right|$$

$$= \int \frac{dt}{t\,(1-t^2)} = \int \frac{dt}{t\,(1-t)(1+t)} \overset{c}{=} \ln|t| \mp \tfrac{1}{2} \ln|1-t| \mp \tfrac{1}{2} \ln|1+t|$$

$$\frac{A}{t} + \frac{B}{1-t} + \frac{C}{1+t} = \frac{1}{\dots} \left(A(1-t^2) + B(t+t^2) + C(t-t^2) \right)$$

$$= \frac{1}{\dots} \left(t^2 \cdot (-A + B - C) + t(B+C) + A \right)$$

$$\Rightarrow \underline{A = 1}, \quad B + C = 0, \quad -A + B - C = 0$$

$$B - C = 1$$

$$2B = 1, \quad \underline{B = \tfrac{1}{2}}, \quad \underline{C = -\tfrac{1}{2}}$$

$$= \ln|t| - \tfrac{1}{2} \ln|1-t^2| = \ln\left|\frac{t}{\sqrt{1-t^2}}\right| = \ln|\operatorname{tg} x|$$

<u>Tj.:</u> $\ln|y| = \ln|\operatorname{tg} x| + C$

$$|y| = e^C \cdot |\operatorname{tg} x|$$

$$y(x) = \pm e^C \operatorname{tg} x = K \cdot \operatorname{tg} x, \quad K \in \mathbb{R} \setminus \{0\}$$

<u>+S.Ř.:</u> $y(x) = K \cdot \operatorname{tg} x$, $K \in \mathbb{R}$ ~~$\setminus\{0\}$~~. $\quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi$

(lze lepit ve bodech $\{k\pi : k \in \mathbb{Z}\}$. $\qquad (k \in \mathbb{Z})$

(2.) $y' = y\cdot\sqrt{1-y^2} = y\sqrt{1-y}\cdot\sqrt{1+y}$; $y \in [-1,1]$

S.Ř.: $y\equiv 0$, $y \equiv 1$, $y \equiv -1$.

$[\ -1 \ \overset{-}{\vdash} \ 0 \ \overset{+}{\vdash} \ 1\ ]$.

Lepení: $g(y) = y\sqrt{1-y^2}$

- na 0: $\int_0^{0+\varepsilon} \frac{dy}{g(y)}$ srovnáme s $\int \frac{1}{y}$:

$$\lim_{y\to 0_+} \frac{\frac{1}{y\sqrt{1-y^2}}}{\frac{1}{y}} = \lim_{y\to 0_+} \frac{1}{\sqrt{1-y^2}} = \frac{1}{\sqrt{1-0^2}} = 1 \in (0,\infty)$$

$\int_0^{\varepsilon} \frac{1}{y}$ diverguje $\Rightarrow \int_0^{\varepsilon} \frac{1}{g}$ D. nelze lepit na 0 shora.

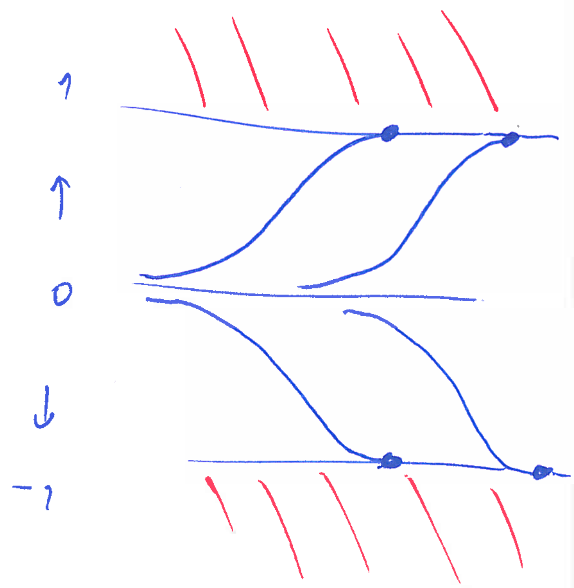
zdola: analogicky, nelze.

- na 1: $\int_{1-\varepsilon}^{1} \frac{1}{g}$ srovnáme s $\int_{1-\varepsilon}^{1} \frac{dy}{\sqrt{1-y}}$, který konv.

$$\lim_{y\to 1_-} \frac{\frac{1}{g(y)}}{\frac{1}{\sqrt{1-y}}} = \lim_{y\to 1_-} \frac{1}{y\sqrt{1+y}} = \frac{1}{1\cdot\sqrt{1+1}} = \frac{1}{\sqrt{2}} \in (0,\infty)$$

Tedy na 1 lze lepit.

- na -1: lze lepit (analog. jako 1)



③ $\cancel{x}y' + 2xy = 2x^3$ metodou IF: $p(x) = 2x$

$(y e^{x^2})' = 2x^3 e^{x^2}$ $P(x) = x^2$, IF: $e^{P(x)} = e^{x^2}$

$$\int 2x^3 e^{x^2}\,dx = \left| \begin{array}{l} t = x^2 \\ dt = 2x\,dx \end{array} \right| = \int t e^t\,dt \overset{P.P.}{=} te^t - \int 1 \cdot e^t\,dt$$

$$\overset{c}{=} te^t - e^t = e^t(t-1) = e^{x^2}(x^2-1)$$

Tedy:

$$y e^{x^2} = e^{x^2}(x^2-1) + C$$

$$y(x) = x^2 - 1 + C e^{-x^2}$$

$$2 = y(0) = 0 - 1 + C \cdot e^0 = C - 1 \implies C = 3$$

Hledané řešení (splňující P.P.) je

$$y(x) = x^2 - 1 + 3e^{-x^2}, \quad x \in \mathbb{R}.$$