1. $y' \cdot \sin x \cdot \cos x - y = 0$ S.Ř. $y \equiv 0$

$y' \cdot \sin x \cos x = y$ $[y \neq 0]$

$\frac{y'}{y} = \frac{1}{\sin x \cdot \cos x}$ $\ln|y| = \ln|\operatorname{tg} x| + \tilde{C}$

$|y| = |\operatorname{tg} x| \cdot e^{\tilde{C}}$

$y = \pm C \cdot \operatorname{tg} x$

$C \in \mathbb{R}$

(lze lepit)

$$\int \frac{dx}{\sin x \cos x} = \int \frac{\cos x \, dx}{\sin x (1 - \sin^2 x)} \leftarrow \begin{bmatrix} z = \sin x \\ dz = \cos x \, dx \end{bmatrix} = \int \frac{dz}{z(1-z^2)}$$

$$\frac{1}{z(1-z)(1+z)} = \frac{A}{z} + \frac{B}{1-z} + \frac{C}{1+z} = \frac{1}{\cdots}\left(A(1-z^2) + B(z+z^2) + C(z-z^2)\right)$$

$\underline{A = 1}$ $B + C = 0$ $-A + B - C = 0$

$B - C = 1$ $2B = 1$ $\underline{B = \frac{1}{2}}$ $\underline{C = -\frac{1}{2}}$

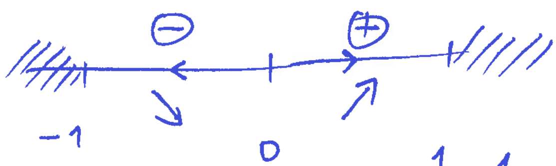
$$= \int \frac{dz}{z} + \frac{1}{2}\int \frac{dz}{1-z} - \frac{1}{2}\int \frac{dz}{1+z} \stackrel{c}{=} \ln|\sin x| - \frac{1}{2}\ln|1 - \sin x| - \frac{1}{2}\ln|1 + \sin x|$$

$$= \ln|\sin x| - \ln\left(|1 - \sin^2 x|\right)^{\frac{1}{2}} = \ln|\sin x| - \ln|\cos x| = \ln|\operatorname{tg} x|$$

Zk: $\left(\ln|\operatorname{tg} x|\right)' = \frac{1}{|\operatorname{tg} x|} \cdot \operatorname{sgn}(\operatorname{tg} x) \cdot \frac{1}{\cos^2 x} = \frac{1}{\frac{\sin x}{\cos x} \cdot \cos^2 x} = \frac{1}{\sin x \cos x}$ ✓

2. $y' = \underbrace{y\sqrt{1-y^2}}_{=:g(y)}$ S.Ř. $y \equiv 0 \,;\, 1 \,;\, -1$

$D_g = \{y : 1 - y^2 \geq 0\} = [-1, 1] \subseteq \mathbb{R}$



Lepení: na 0: $\int_0^{\frac{1}{2}} \frac{1}{g(y)}$ porovnáme s $\frac{1}{y^k}$:

$$\lim_{y \to 0} \frac{\frac{1}{g(y)}}{\frac{1}{y}} = \lim_{y \to 0} \frac{1}{\sqrt{1-y^2}} = 1 \in (0, \infty) \rightarrow \ldots \Rightarrow \int_0^{\frac{1}{2}} \frac{1}{g}, \int_{-\frac{1}{2}}^{0} \frac{1}{g}$$

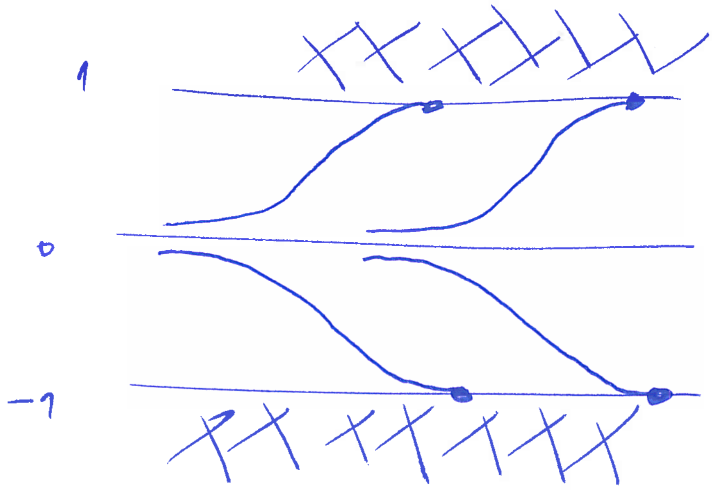
D. $\Rightarrow$ nelze lepit.

na 1: Srovnáme s $\frac{1}{\sqrt{1-y}}$:

$$\lim_{y\to 1_-} \frac{\frac{1}{g(y)}}{\frac{1}{\sqrt{1-y}}} = \lim \frac{\sqrt{1-y}}{y\sqrt{1-y}\cdot\sqrt{1+y}} = \lim_{y\to 1_-} \frac{1}{y\sqrt{1+y}} = \frac{1}{\sqrt{2}} \in (0,\infty).$$

ale $\int_{\frac{1}{2}}^{1} \frac{dy}{\sqrt{1-y}} = \ldots = \int_0^{\frac{1}{2}} \frac{dy}{\sqrt{y}}$ k., tedy lze lepit

na -1: analog.



③

$$y' + 2xy = 2x^3 \qquad y(0) = 2$$

I.F. $p(x) = 2x$ $\quad P(x) = x^2$ $\quad$ I.F. $:= e^{x^2}$

$$y'e^{x^2} + e^{x^2}2x\,y = e^{x^2}2x^3$$

$$(ye^{x^2})' = e^{x^2}\cdot 2x^3$$

$\int e^{x^2}2x^3\,dx = \int e^{x^2}\cdot 2x\cdot x^2\,dx$

$\leftarrow \begin{bmatrix} t = x^2 \\ dt = 2x\,dx \end{bmatrix} = \int e^t\cdot t\,dt \overset{\text{P.P.}}{=}$

$te^t - \int e^t\,dt \overset{c}{=} e^t(t-1)$

$$y(x)e^{x^2} = e^{x^2}(x^2-1) + C$$

$$y(x) = x^2 - 1 + Ce^{-x^2}, \quad C\in\mathbb{R},\ x\in\mathbb{R}$$

$$2 = y(0) = C - 1 \qquad C = 3$$

$$\boxed{y(x) = x^2 - 1 + 3e^{-x^2}}$$