1. $\left[y' = x\cdot\sqrt{y}\right.$, $\left. y(0) = 2.\right]$ $\Rightarrow y \geq 0$

STAC. Ř. : $y \equiv 0$ na $\mathbb{R}$. nyní nechť $y \neq 0$:

$$\frac{y'}{\sqrt{y}} = x \quad , \quad 2\sqrt{y} = \frac{x^2}{2} + C \ , \quad C \in \mathbb{R}$$

$$\left(\Rightarrow \frac{x^2}{2} + C > 0 \ , \ \underline{\cancel{\tfrac{x^2}{4}}\ x^2 > -2C} \right)$$

$$\sqrt{y} = \frac{\frac{1}{2}x^2 + C}{2} \quad , \quad y = \frac{1}{4}\left(\frac{x^2}{2} + C\right)^2$$

- $C > 0$ ... $x^2 > -2C$ pro $x \in \mathbb{R}$
- $C \geq 0$ ... vzorec platí pro $x \in (\sqrt{-2C}, \infty)$ a pro $x \in (-\infty, -\sqrt{-2C})$

možnosti max. řešení:

1) pro $C > 0$: $y_1(x) = \frac{1}{4}(x^2/2 + C)^2$, $x \in \mathbb{R}$

pro $C \leq 0$:

2) $$y_2(x) = \begin{cases} 0 & , x \in (-\infty, \sqrt{-2C}] \\ \frac{1}{4}\left(\frac{x^2}{2} + C\right)^2 & , x > \sqrt{-2C} \end{cases}$$

3) $$y_3(x) = \begin{cases} \frac{1}{4}\left(\frac{x^2}{2} + C\right)^2 & , x < -\sqrt{-2C} \\ 0 & , x \geq -\sqrt{-2C} \end{cases}$$

4) $$y_4(x) = \begin{cases} \frac{1}{4}\left(\frac{x^2}{2} + C\right)^2 & , x < -\sqrt{-2C} \\ 0 & , x \in [-\sqrt{-2C}, \sqrt{-2C}] \\ \frac{1}{4}\left(\frac{x^2}{2} + C\right)^2 & , x > \sqrt{-2C} \end{cases}$$

P.P.: jedině y_1!

Tedy $2 = y_1(0) = \frac{1}{4}\left(\frac{0^2}{2} + C\right)^2 = \frac{1}{4}C^2$

$\Rightarrow C = \pm\sqrt{8}$,

ale $C > 0$, tj $\underline{\underline{C = \sqrt{8}}}$

Tedy řešení je
$$y(x) = \frac{1}{4}\left(\frac{x^2}{2} + \sqrt{8}\right)^2.$$

② $y' = (1-\cos y)\cdot\sqrt[5]{y^3-y^2+y-1}$. Kvalitativně:

STAC. ŘEŠENÍ: $1-\cos y = 0 \iff \underline{y = 2k\pi}$, $k\in\mathbb{Z}$.

$y^3-y^2+y-1 = 0$ ... uhodneme kořen $y=1$

$(y^3-y^2+y-1):(y-1) = \dots$ snadno $\dots\ y^2+1$.

Tedy $y^3-y^2+y-1 = (y-1)(y^2+1) = 0 \iff \underline{y=1}$

Monotonie:

$$\begin{array}{ccccccccccccc} \dots & & - & & - & & - & & + & & + & & + \quad \dots \\ & -4\pi & & -2\pi & & 0 & & 1 & & 2\pi & & 4\pi & \end{array}$$

LEPENÍ: na 1: • $\int_1^2 \frac{1}{g}$ srovnáme: $\frac{1}{\sqrt[5]{y-1}}$

$$\lim_{y\to1_+}\frac{\frac{1}{g(y)}}{\frac{1}{\sqrt[5]{y-1}}} = \lim_{y\to1_+}\frac{\sqrt[5]{y-1}}{(1-\cos y)\sqrt[5]{(y-1)(y^2+1)}} = \lim_{y\to1_+}\frac{1}{(1-\cos y)\sqrt[5]{y^2+1}} \in (0,\infty)$$

Takže $\int_1^2\frac{1}{g}$ K. $\iff \int_1^2\frac{dy}{\sqrt[5]{y-1}}$ K, ten ale K. $\Rightarrow$ lze lepit.

• Analogicky zespoda – $\int_{\frac{1}{2}}^1\frac{1}{g}$ K.

na $2k\pi$: Srovnáme s $\frac{1}{(y-2k\pi)^2}$:

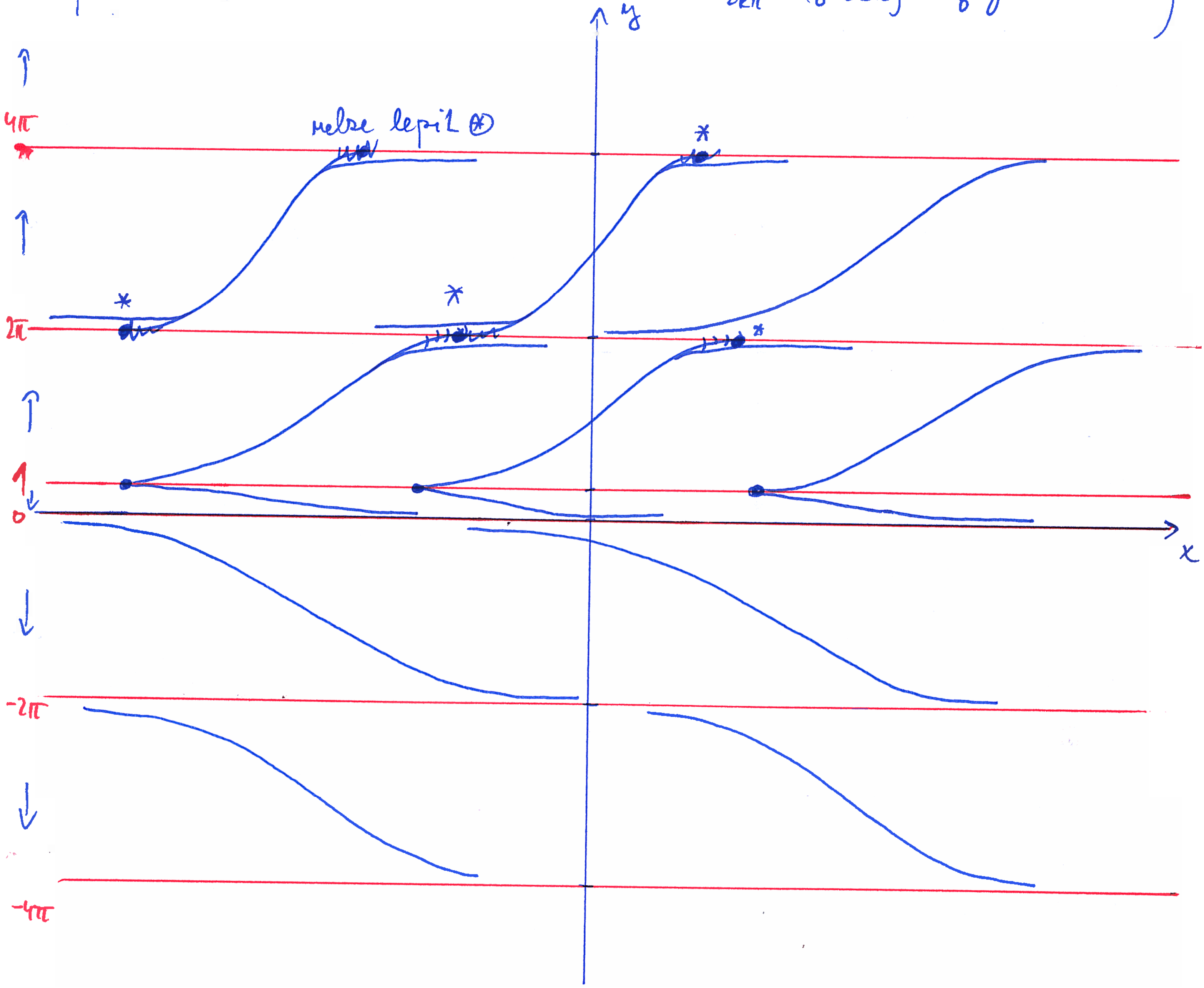
$$\lim_{y\to2k\pi}\frac{\frac{1}{g}}{\frac{1}{(y-2k\pi)^2}} = \lim_{y\to2k\pi}\frac{(y-2k\pi)^2}{1-\cos y}\cdot\lim_{y\to2k\pi}\frac{1}{\sqrt[5]{y^3-y^2+y-1}} =$$

$$\underbrace{\left((2k\pi)^3-(2k\pi)^2+2k\pi-1\right)^{-\frac{1}{5}}}_{=:K\neq0}\cdot\lim_{y\to2k\pi}\frac{(y-2k\pi)^2}{1-\cos y} = K\cdot\lim_{\tilde y\to0}\frac{\tilde y^2}{1-\cos(\tilde y+2k\pi)} =$$

$$= K\cdot\lim_{\tilde y\to0}\frac{\tilde y^2}{1-\cos\tilde y} = 2K\in\mathbb{R}\setminus\{0\},$$

takže $\int_{2k\pi}^{2k\pi+\frac{1}{2}} \frac{1}{g}$ D. a $\int_{2k\pi-\frac{1}{2}}^{2k\pi} \frac{1}{g}$ D.

Lepil tedy nelze. $\searrow$ (Protože $\int_{2k\pi}^{2k\pi+\frac{1}{2}} \frac{dy}{(y-2k\pi)^2} = \int_0^{\frac{1}{2}} \frac{1}{y^2}dy = \infty$)



(3) $$y' - \frac{2\sin x}{\cos x}\, y = \frac{1}{\sin^2 x} \qquad x \in \left(\frac{k\pi}{2}, \frac{(k+1)\pi}{2}\right), \; k \in \mathbb{Z}.$$

INTEGRAČNÍ FAKTOR: $p(x) = -\dfrac{2\sin x}{\cos x}$

$$P(x) \overset{c}{=} -\int \frac{2\sin x}{\cos x}\,dx = 2\int \frac{-\sin x}{\cos x}\,dx \overset{c}{=} 2\ln|\cos x| = \ln \cos^2 x$$

I.F.: $e^{\ln \cos^2 x} = \cos^2 x.$

$$y' \cdot \cos^2 x + (-2\sin x \cos x) \cdot y = \frac{\cos^2 x}{\sin^2 x}$$

$$(y \cdot \cos^2 x)' = \frac{\cos^2 x}{\sin^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x} = \frac{1-\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

$$\int \frac{dx}{\sin^2 x} = -\int \frac{-1}{\sin^2 x}\,dx \overset{c}{=} -\cot g\, x$$

Tedy:

$$y \cdot \cos^2 x = -\cot g\, x - x + C, \quad C \in \mathbb{R}$$

$$y(x) = \frac{-1}{\cos x \cdot \sin x} - \frac{x}{\cos^2 x} + \frac{C}{\cos^2 x}, \quad C \in \mathbb{R},$$

$$x \in \left(\frac{k\pi}{2}, \frac{(k+1)\pi}{2}\right), \; k \in \mathbb{Z}.$$