1. $\frac{y'}{y+2} = xy$ $\quad [y \neq -2]$, STAC. Ř. $[y \equiv 0]$

$y' = xy(y+2)$ $\quad \underline{y \neq 0}$

$\frac{y'}{y(y+2)} = x$

$\left[\text{L.S.}: \int \frac{dy}{y(y+2)} = \frac{1}{2}\int \frac{dy}{y} - \frac{1}{2}\int \frac{dy}{y+2} = \right.$

$\left[\frac{1}{y(y+2)} = \frac{A}{y} + \frac{B}{y+2} = \frac{Ay + 2A + By}{y(y+2)} \right.$

$\left.\begin{aligned} A + B &= 0 \\ 2A &= 1 \end{aligned}\right\} \Rightarrow \underline{A = \frac{1}{2}}, \quad \underline{B = -\frac{1}{2}} \Big]$

$\left. \overset{c}{=} \frac{1}{2}(\ln|y| - \ln|y+2|) = \frac{1}{2}\ln\left|\frac{y}{y+2}\right| \right]$

$\frac{1}{2}\ln\left|\frac{y}{y+2}\right| = \frac{1}{2}x^2 + C_1$

$\ln\left|\frac{y}{y+2}\right| = x^2 + C_2$

$\left|\frac{y}{y+2}\right| = e^{C_2} \cdot e^{x^2}$

$\frac{y}{y+2} = \pm e^{C_2} \cdot e^{x^2} =: K \cdot e^{x^2}, \quad K \in \mathbb{R}\setminus\{0\},$

zahrneme $y \equiv 0$ ... $K \in \mathbb{R}$.

$y(1 - K \cdot e^{x^2}) = 2Ke^{x^2}$

$y(x) = \frac{2Ke^{x^2}}{1 - Ke^{x^2}}$,

$1 - Ke^{x^2} \neq 0$

$Ke^{x^2} \neq 1$

$e^{x^2} \neq \frac{1}{K}$

$x^2 \neq \ln\left(\frac{1}{K}\right)$

$x \neq \pm\sqrt{-\ln K}$

problém pouze pro $K \leq 1$

$K \leq 0$ ... $x \in \mathbb{R}$

$K \in (0, 1)$ ... $x \in \begin{cases} \left(-\infty, -\sqrt{\ln \frac{1}{K}}\right) \\ \left(-\sqrt{\ln \frac{1}{K}}, \sqrt{\ln \frac{1}{K}}\right) \\ \left(\sqrt{\ln \frac{1}{K}}, \infty\right) \end{cases}$

$K = 1$ ... $x \in \begin{cases} (-\infty, 0) \\ (0, \infty) \end{cases}$

$K > 1$ ... $x \in \mathbb{R}$.

② $y' = \sqrt{y^2-2}\cdot(y^2-9)$ ... S.Ř.: $\pm\sqrt{2}$, ± 3

monotonie:

$(-\infty,-3)$	$(-3,-\sqrt{2})$	$(-\sqrt{2},0)$	$(0,\sqrt{2})$	$(\sqrt{2},3)$	$(3,\infty)$
$+$	$-$	$\emptyset$	$\emptyset$	$-$	$+$

Lepeníčko: • na $\sqrt{2}$ shora: $\int_{\sqrt{2}}^{2}\frac{1}{g} = \int_{\sqrt{2}}^{2}\frac{1}{\sqrt{y^2-2}\,(y^2-9)}\,dy$

srovnáme s $\frac{1}{\sqrt{y-\sqrt{2}}}$:

$$\lim_{y\to\sqrt{2}+}\frac{\frac{1}{\sqrt{y^2-2}\,(y^2-9)}}{\frac{1}{\sqrt{y-\sqrt{2}}}} = \lim_{y\to\sqrt{2}+}\frac{\sqrt{y-\sqrt{2}}}{\sqrt{y-\sqrt{2}}\cdot\sqrt{y+\sqrt{2}}\cdot(y^2-9)} =$$

$$= \lim_{y\to\sqrt{2}+}\frac{1}{\sqrt{y+\sqrt{2}}\,(y^2-9)} = \frac{1}{\sqrt{2\sqrt{2}}\cdot(2-9)} \in (-\infty,0)$$

Podle LSK tedy: $\int_{\sqrt{2}}^{2}\frac{1}{g}$ K $\iff$ $\int_{\sqrt{2}}^{2}\frac{dy}{\sqrt{y-\sqrt{2}}}$ K, ale ten K.

Tedy lepit lze (shora; zdola „nic není").

• na $-\sqrt{2}$ „zdola": analog., lepit lze.

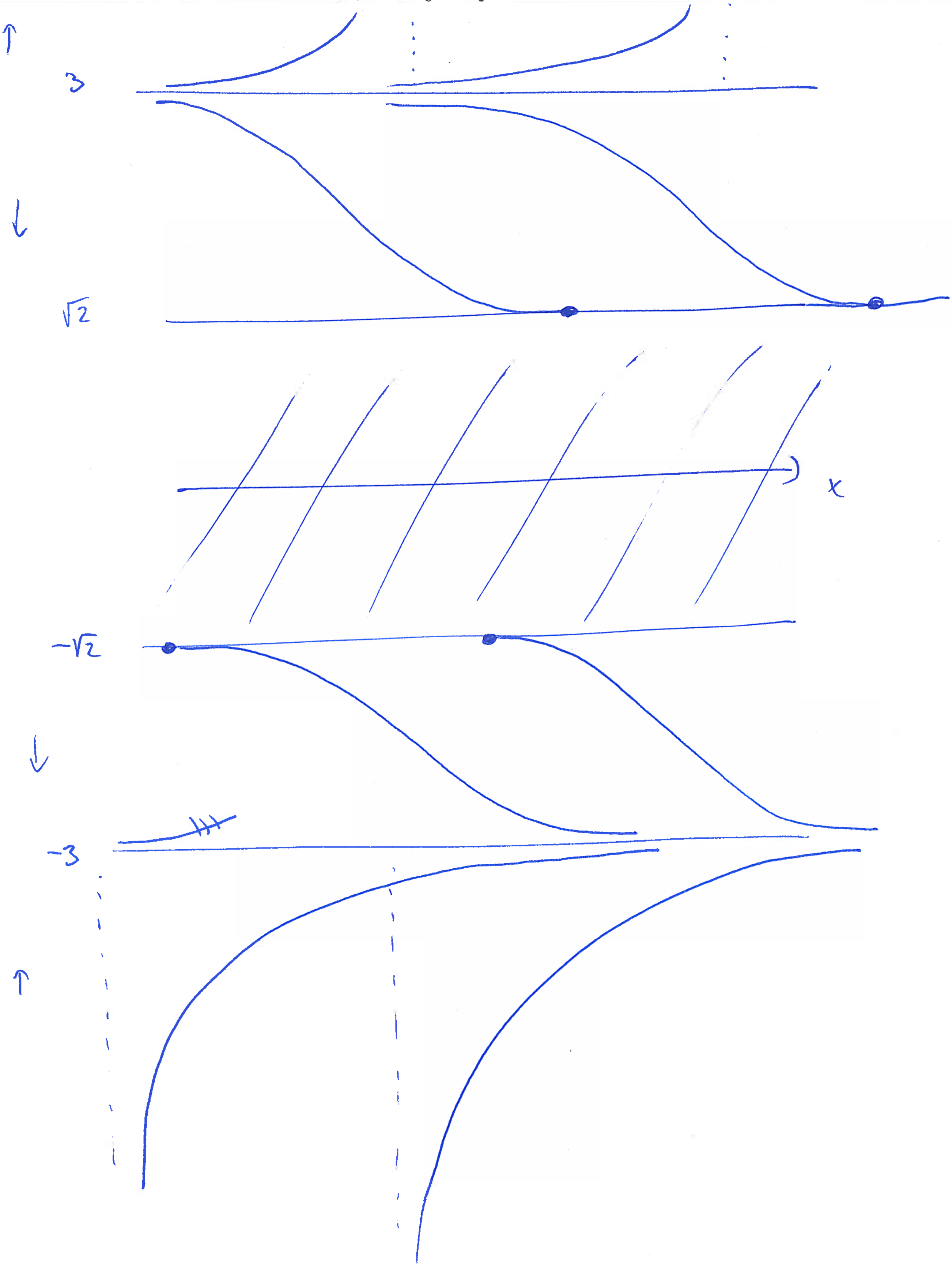
• na 3: $\int_2^3\frac{1}{g}$ resp. $\int_3^4\frac{1}{g}$ ... srovnáme s $\frac{1}{y-3}$:

$$\lim_{y\to 3}\frac{\frac{1}{g(y)}}{\frac{1}{y-3}} = \lim_{y\to 3}\frac{1}{\sqrt{y^2-2}\,(y+3)} = \frac{1}{\sqrt{9-2}\cdot 6} \in (0,\infty).$$

Tedy podle LSK: $\int_2^3\frac{1}{g}$ (resp. $\int_3^4\frac{1}{g}$) K. $\iff$

$\int_2^3\frac{dy}{y-3}$ (resp. $\int_3^4\frac{dy}{y-3}$) K., ale

ten D., takže lepit nelze.


3
$\sqrt{2}$
x
$-\sqrt{2}$
-3

③ $y' + 3x^2 y - e^{x-x^3}\cos x = 0$

$y' + 3x^2 y = e^{x-x^3}\cos x$

I.F. : $p(x) = 3x^2$, $P(x) = x^3$, I.F. : e^{x^3}

$e^{x^3} y' + e^{x^3} 3x^2 y = e^x \cos x$

$\left(e^{x^3} y\right)' = e^x \cos x \quad / \int \cdots dx$

$\int \overset{\downarrow}{e^x} \overset{\uparrow}{\cos x}\, dx = e^x \sin x - \int \overset{\downarrow}{e^x} \overset{\uparrow}{\sin x}\, dx =$

$= e^x \sin x - \left(-e^x \cos x - \int e^x(-\cos x)\,dx\right) =$

$= \underline{e^x \sin x + e^x \cos x} - \int e^x \cos x\, dx \implies$

$2\int e^x \cos x\, dx \overset{c}{=} \ldots$, a tedy $\int e^x \cos x\,dx \overset{c}{=} \frac{e^x}{2}(\sin x + \cos x)$

$e^{x^3} y = \frac{e^x}{2}(\sin x + \cos x) + C, \quad C \in \mathbb{R}$

$y(x) = \frac{e^{x-x^3}}{2}(\sin x + \cos x) + C \cdot e^{-x^3},$

$x \in \mathbb{R} \qquad (C \in \mathbb{R}).$