① $y' = \dfrac{e^{y}}{x(1+x^2)}$ $\qquad$ $x \neq 0$, $\quad$ STAC. Ř. není.

$$e^{-y}\, y' = \frac{1}{x(1+x^2)} \quad \dots$$

$$\left[= \frac{A}{x} + \frac{Bx+C}{1+x^2} = \frac{Ax^2+A+Bx^2+Cx}{x(1+x^2)} \right.$$

$$\Rightarrow A+B=0, \quad C=0, \quad A=1, \quad B=-1 \Big]$$

$$-e^{-y} \overset{c}{=} \int \left(\frac{1}{x} - \frac{x}{1+x^2}\right) dx$$

$$-e^{-y} = \ln|x| - \frac{1}{2}\ln(1+x^2) + C$$

$$\int \frac{-x}{1+x^2}\,dx = -\frac{1}{2}\int \frac{2x}{1+x^2}\,dx \overset{c}{=} -\frac{1}{2}\ln|1+x^2|$$

$$+e^{-y} = -\left(\ln\frac{|x|}{\sqrt{1+x^2}} + C\right) \Longrightarrow -C - \ln\frac{|x|}{\sqrt{1+x^2}} > 0$$

$$-y = \ln\left(\ln\frac{\sqrt{1+x^2}}{|x|} - C\right)$$

$$\boxed{\ln\frac{|x|}{\sqrt{1+x^2}} < -C}$$

Pro $C \geq 0$ automaticky $x \in \mathbb{R}\setminus\{0\}$

$$\frac{|x|}{\sqrt{1+x^2}} < e^{-C}$$

$$\frac{x^2}{1+x^2} < e^{-2C}$$

$$x^2 < e^{-2C}(1+x^2)$$

$$x^2(1-e^{-2C}) < e^{-2C}$$

$1-e^{-2C} > 0$

$e^{-2C} < 1$

$\boxed{C>0}$

$$\Longrightarrow x^2 < \frac{e^{-2C}}{1-e^{-2C}}$$

$$x \in \left(-\sqrt{\frac{e^{-2C}}{1-e^{-2C}}}, \sqrt{\frac{e^{-2C}}{1-e^{-2C}}}\right), \quad C>0$$

$$y(x) = -\ln\left(\ln\frac{\sqrt{1+x^2}}{|x|} - C\right)$$

$C \leq 0$: $\quad x \in (-\infty, 0)$; $(0, \infty)$

$C > 0$: $\quad x \in \left(-\sqrt{\frac{e^{-2C}}{1-e^{-2C}}}, 0\right)$; $\left(0, \sqrt{\frac{e^{-2C}}{1-e^{-2C}}}\right)$.

(2.) $y' = \underbrace{\frac{\ln(y+6)\sqrt[3]{y}}{y^5+32}}_{=:\, g(y)}$

[<u>STAC. Ř.</u>: $\ln(y+6)=0 \iff y+6=1$; $\underline{y \equiv -5}$, $\underline{y \equiv 0}$]

<u>Monotonie</u>:

[$y^5 \neq \overset{-2^5}{\overset{\shortparallel}{-32}} \iff \underline{y \neq -2}$]

[$y+6>0 \iff \underline{y > -6}$]

$-\quad +\quad -\quad +$

$[-6 \quad -5 \quad -2 \quad 0$

<u>Lepení</u>: • „na -5": $\int_{-5}^{-4} \frac{1}{g}$ … „srovnej s $\frac{1}{\ln(y+6)}$":

$$\lim_{y\to -5_+} \frac{\frac{1}{g}}{\frac{1}{\ln(y+6)}} = \lim_{y\to -5_+} \frac{\ln(y+6)\,(y^5+32)}{\ln(y+6)\sqrt[3]{y}} = \frac{(-5)^5+32}{\sqrt[3]{-5}} \in (0,\infty),$$

tedy podle LSK: $\int_{-5}^{-4}\frac{1}{g}$ K. $\iff \int_{-5}^{-4}\frac{dy}{\ln(y+6)}$ K.

Ale $\ln(y+6)$ je spoj. na $[-5,-4]$, kladná na $(-5,-4]$,
$\ln(-5+6)=0$, $\left(\ln(y+6)\right)'\Big|_{y=-5} = \frac{1}{y+6}\Big|_{y=-5} = 1 \in \mathbb{R}$.

Podle Lemmatu (13) D. $\Rightarrow$ <u>nelze lepit</u>. (shora)

Podobně zdola.

• „na 0": $\int_0^1 \frac{1}{g}$, „srovnej s $\frac{1}{\sqrt[3]{y}}$":

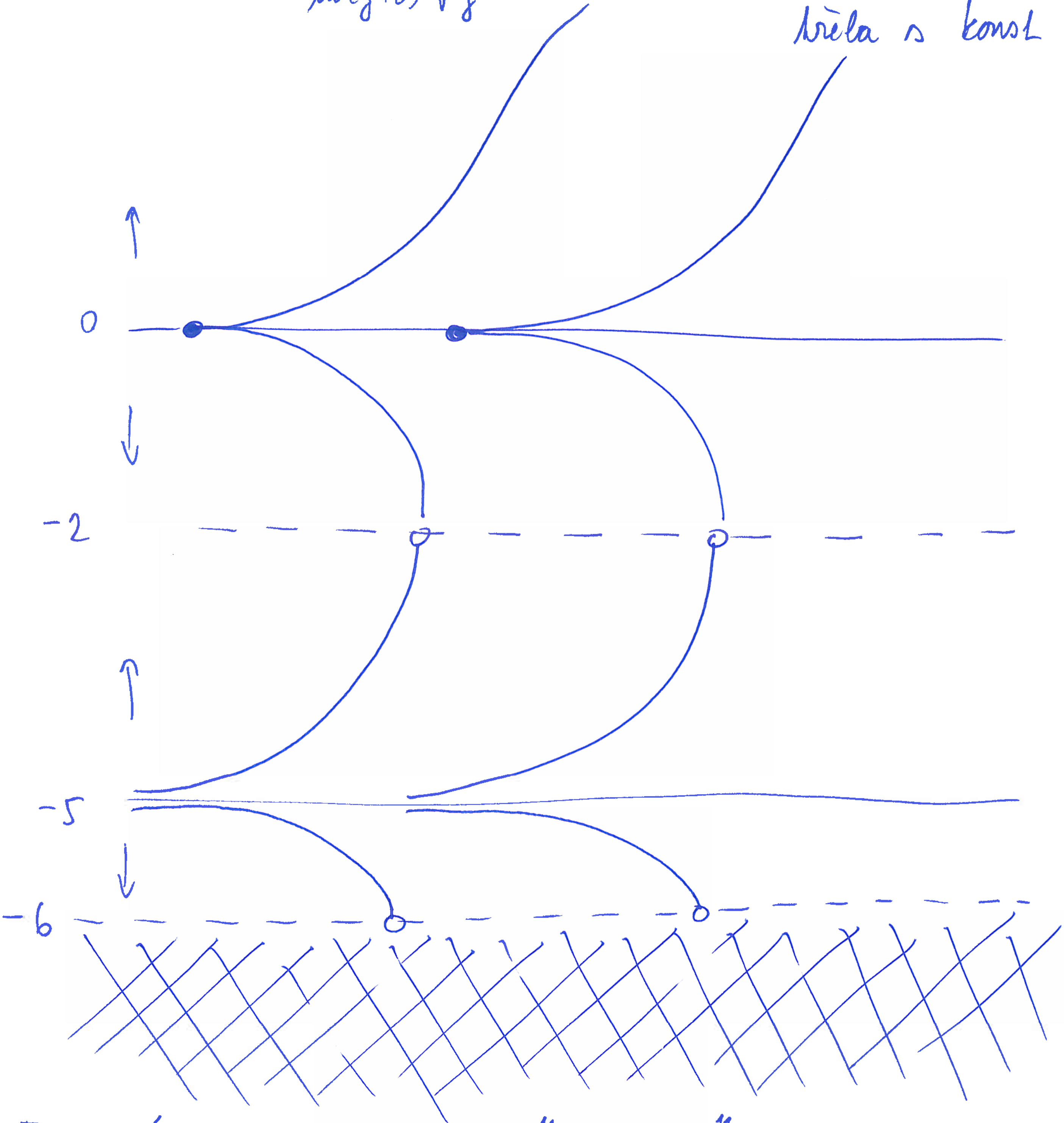
$$\lim_{y\to 0_+} \frac{\frac{1}{g}}{\frac{1}{\sqrt[3]{y}}} = \lim_{y\to 0_+} \frac{y^5+32}{\ln(y+6)} = \frac{32}{\ln 6} \in (0,\infty).$$ Tedy LSK:

$\int_0^1 \frac{1}{g}$ K. $\iff \int_0^1 \frac{1}{\sqrt[3]{y}}\,dy$ K., ale ten K. $\Rightarrow$ <u>lze lepit</u>.

Podobně zdola.

• „Svislé asymptoty – v ∞": <u>Nejsou</u>:

$$\int_{10}^{\infty} \frac{1}{g} = \int_{10}^{\infty} \frac{y^5+32}{\ln(y+6)\sqrt[3]{y}}\, dy = \infty$$ … snadno srovnáním třeba s konst. 1.



PRO ZAJÍMAVOST:

• „Chování u -6:" $$\int_{-6}^{-\frac{11}{2}} \frac{1}{g} = \int_{-6}^{-\frac{11}{2}} \frac{y^5+32}{\ln(y+6)\sqrt[3]{y}}\, dy \quad K. \, :$$

Srovnej s 1: $$\lim_{x\to -6+} \frac{\frac{1}{g}}{1} = \lim_{x\to -6+} \frac{y^5+32}{\ln(y+6)\sqrt[3]{y}} = \text{typ}_{x\to -6} \frac{(-6)^5+32}{(-\infty)\cdot\sqrt[3]{-6}} = 0$$

Tedy $\left[\int_{-6}^{-\frac{11}{2}} 1 \;\; K. \Rightarrow \int \frac{1}{g} \;\; K.\right]$, tedy „dosáhne" hladiny -6 v kon. čase."

3. • $y' - \frac{2x}{x^2+1} y = x^2$, $y(1) = \pi$

$$\left[\begin{array}{l} p(x) = \frac{-2x}{x^2+1}, \quad P(x) \overset{c}{=} -\int \frac{2x\,dx}{x^2+1} \overset{c}{=} -\ln(x^2+1) \\ \text{I.F.}: \quad e^{-\ln(x^2+1)} = \frac{1}{x^2+1}. \end{array} \right]$$

$$y' \cdot \frac{1}{x^2+1} - y \frac{2x}{(x^2+1)^2} = \frac{x^2}{x^2+1},$$

$$\left(y \cdot \frac{1}{x^2+1} \right)' = \frac{x^2+1-1}{x^2+1}, \quad \text{tj.} \quad y \cdot \frac{1}{x^2+1} \overset{c}{=} \int \left(1 - \frac{1}{x^2+1}\right) dx$$

$$y \cdot \frac{1}{x^2+1} = x - \operatorname{arctg} x + C$$

$$y(x) = (x - \operatorname{arctg} x + C)(x^2+1), \quad x \in \mathbb{R} \quad (C \in \mathbb{R})$$

$$\pi = y(1) = (1 - \operatorname{arctg} 1 + C) \cdot 2 = \left(1 - \tfrac{\pi}{4} + C\right) \cdot 2$$

$\frac{\pi}{2} = 1 - \frac{\pi}{4} + C \implies C = \frac{3\pi}{4} - 1.$ Tedy řešení P.P.:

$$y(x) = \left(x - \operatorname{arctg} x + \frac{3\pi}{4} - 1\right)(x^2+1), \quad x \in \mathbb{R}.$$

• $y^{(4)} - y = 0$ ... CH.P. $\lambda^4 - 1 = (\lambda^2-1)(\lambda^2+1)$

<u>KOŘENY CH. P.</u>: $-1, 1, -i, i$

F.S. $:= \{ e^{-t}, e^{t}, \cos t, \sin t \}$

Obecné max. řešení je tedy tvaru

$$y(x) = C_1 e^{-t} + C_2 e^{t} + C_3 \cos t + C_4 \sin t, \quad t \in \mathbb{R}$$

$(C_1, C_2, C_3, C_4 \in \mathbb{R})$.