20/1/j): $\displaystyle\int_0^\infty \frac{x^p}{1+x^q}\,dx \qquad (p,q\in\mathbb{R}).$

„U nuly": (a) pro $q\geq 0$ srovnáme s $g(x)=x^p$. Pak:

$$\lim_{x\to0_+}\frac{f(x)}{g(x)}=\lim_{x\to0_+}\frac{\frac{x^p}{1+x^q}}{x^p}=\lim_{x\to0_+}\frac{1}{1+x^q}=\begin{cases}\frac12, & q=0\\ 1, & q>0\end{cases}$$

$1\in(0,\infty)$; LSK: $\int_0^1 f$ K $\overset{(LSK)}{\iff}\int_0^1 g$ K $\iff -p<1$

resp. $\frac12\in$

(b) $q<0$: srovnáme s

$$g(x)=\frac{x^p}{x^q}=x^{p-q}.\ \text{LSK:}$$

$$\lim_{x\to0_+}\frac{f(x)}{g(x)}=\lim_{x\to0_+}\frac{\frac{x^p}{1+x^q}}{\frac{x^p}{x^q}}=\lim_{x\to0_+}\frac{x^q}{1+x^q}=$$

$$=\lim_{x\to0_+}\frac{\cancel{x^q}}{\cancel{x^q}\cdot\left(1+\frac{1}{\underbrace{x^q}_{\to\infty}}\right)}=\lim_{x\to0_+}\frac{1}{1+\frac{1}{x^q}}=1$$

LSK: $\int_0^1 f$ K $\iff \int_0^1 g \iff -(p-q)<1$

„U nekonečna": (a) $q\geq 0$ srovnáme s

$$g(x)=\frac{x^p}{x^q}=x^{p-q}$$

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=\dots=\lim_{x\to\infty}\frac{x^q}{1+x^q}=\begin{cases}\frac12, & \text{pokud } q=0\\ 1, & q>0\end{cases}$$

Tedy (LSK): $\int_1^\infty f$ K $\iff \int_1^\infty g$ K $\iff -(p-q)>1$

(b) $q<0$ srovnáme s x^p: LSK

$$\lim_{x\to\infty}\frac{\frac{x^p}{1+x^q}}{x^p}=\lim_{x\to0}\frac{1}{1+\overbrace{x^q}^{\to0}}=\frac{1}{1+0}=1\in(0,\infty)$$

Tedy $\int_1^\infty f$ K $\iff \int_1^\infty x^p$ K $\iff -p>1$.

Tedy: Pro $q < 0$ $\int_0^\infty f$ k. $\iff$

$\iff p - q > -1 \wedge p < -1$

Pro $q \geq 0$ $\int_0^\infty f$ k. $\iff$

$\iff p > -1 \wedge p - q < -1$

Celkem: $\int_0^\infty f(x)\,dx = \int_0^\infty \frac{x^p}{1+x^q}\,dx$

konverguje $\iff$

$$\big(q < 0 \wedge p \in (q-1, -1)\big) \vee$$
$$\big(q \geq 0 \wedge p \in (-1, q-1)\big).$$

Pozn.: Tuto podmínku nelze splnit pro $q = 0$, v druhé alternativě výše tedy lze psát i $q > 0$.