

FACULTY OF MATHEMATICS AND PHYSICS Charles University

28th Annual Student Conference Week of Doctoral Students

Book of Abstracts

of the

6th Week of Doctoral Students of the School of Mathematics 2019

June 6, 2019



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Preface

In the beginning of 2014, the Management of the Faculty of Mathematics and Physics decided that the traditional conference of PhD students called the WDS (Week of Doctoral Students) would not be organized as an activity of the entire faculty. Instead, the decision as to whether to organize the conference or not was left to the respective Schools (of Computer Science, of Mathematics, and of Physics).

Already for the sixth year since then the School of Mathematics organizes its WDS-M (Week of Doctoral Students of the School of Mathematics, http://www.karlin.mff.cuni.cz/~rokyta/WDS-M/2019/), this time again as a one-day conference, in the framework, and as a continuation of, the (28th) WDS of the Faculty of Mathematics and Physics (http://www.mff.cuni.cz/veda/konference/wds/).

This year, 22 students have registered as active participants to the conference. We believe that this event, which takes place in the "mathematical" Karlín building of the faculty, will attract the attention of the students but also of the broad mathematical audience. We thus encourage all of those interested in the scientific activities of our doctoral students to attend this meeting.

The conference is co-organized by the *School of Mathematics, Faculty of Mathematics and Physics, Charles University*, and *Charles University Chapter of SIAM*.

Prague, June 6, 2019

doc. RNDr. Mirko Rokyta, CSc. Vice-Dean for Mathematics Faculty of Mathematics and Physics Charles University Prague

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MAE Formalization of CSP dichotomy in proof complexity

Contributed talk

Mgr. Azza Gaysin

E-mail: azza.gaysin@gmail.com *Study branch:* 4M1 – Algebra, number theory, and mathematical logic *Year of study:* 1. *Supervisor:* prof. RNDr Jan Krajíček, DrSc.

Abstract

The theory of CSP provides an universal apparatus and a simple formal framework for the representation and solution of a wide range of natural combinatorial problems. In the CSP associated with a finite relational structure A [CSP(A)] the question is if there exists a homomorphism of a given relational structure B to A.

Proof complexity is an area connecting mathematical logic and computational complexity theory, whose main problem is whether the class NP is closed under complementation. The main tool of proof complexity is the set of propositional proof systems, formally, a polynomial-time functions whose range is the set of all propositional tautologies. if A is a formula, and P is a proof system, then any string w such that P(w) = A is called a Pproof of A. We say that some tautology (problem) is easy for proof system iff there is polynomial-size proof of this tautology in this system.

The Dichotomy Conjecture for CSP of Feder and Vardi, that all CSP problems either tractable or NP-complete [T. Feder, M.Y. Vardi, The computational structure of monotone monadic SNP and constraint satisfaction: a study through datalog and group theory, *SIAM J. Comput.* 28 (1998) 57–104.], was recently proved by Zhuk [D. Zhuk A Proof of CSP Dichotomy Conjecture, *IEEE 58th Annual Symposium on Foundations of Computer Science* (FOCS), 2017] and Bulatov [A. Bulatov, Constraint Satisfaction Problems: Complexity and Algorithms, *SIGLOG* News 5(4): 4-24 (2018)]. We try to formalize this result in as weak as possible proof system for better understanding of proof complexity of CSP and of the power of proposition-al proof systems.

Ring epimorphisms

Contributed talk

Mgr. Jakub Kopřiva

E-mail: jakub.kopriva@outlook.com *Study branch:* 4M1 – Algebra, number theory, and mathematical logic *Year of study:* 1. *Supervisor:* doc. RNDr. Jan Šť ovíček, Ph.D.

Abstract

Ring epimorphism is a ring homomorphism $\varphi : A \to B$ such that for any pair of ring homomorphisms $\rho_1, \rho_2 : B \to C$ such that $\rho_1 \varphi = \rho_2 \varphi$, already $\rho_1 = \rho_2$ – this means that, in a way, a subring Im φ controls the whole ring *S*. Contrary to our intuition, ring epimorphisms need not to be surjective (as it is the case for Lie algebras, *B*-modules or topological spaces). An ordinary localization $R \to R_S$ of a commutative ring *R* in a multiplicative set of its elements *S* may serve as an example. Due to being non-trivial and having a neat description in terms of categories of modules, ring epimorphisms are an object of study within the representation theory.

In this talk, we will define ring epimorphisms and examine their properties with regard to examples from commutative algebra and representation theory of finite-dimensional algebras.

Universal quadratic forms over number fields

Contributed talk

Ing. Jakub Krásenský

E-mail: krasensky@karlin.mff.cuni.cz *Study branch:* 4M1 – Algebra, number theory, and mathematical logic *Year of study:* 1. *Supervisor:* Mgr. Vítězslav Kala, Ph.D.

Abstract

Quadratic forms play an important role in many areas of mathematics. In number theory, one classical question about a given positive definite form with integer coefficients concerns its "universality", i.e. whether or not it represents all positive integers. The first important result in this direction is Lagrange's Four squares theorem; further progress was achieved by the likes of Ramanujan and Conway. Universality of quadratic forms over \mathbb{Z} is essentially solved; however, the same question over a ring of integers in a number field is a vivid area of research – also in our faculty. This talk provides an introduction into this topic and briefly touches on recent results by the author, M. Tinková and K. Zemková.

Jacobi-Perron algorithm and indecomposable integers in the simplest cubic fields

Contributed talk

Ing. Magdaléna Tinková

E-mail: tinkova.magdalena@gmail.com *Study branch:* 4M1 – Algebra, number theory, and mathematical logic *Year of study:* 2. *Supervisor:* Mgr. Vítězslav Kala, Ph.D.

Abstract

In this talk, we will examine the particular subset of the ring of algebraic integers of totally real number fields, so-called indecomposable integers. In quadratic fields, we can find all of them using the continued fraction of some numbers. However, we do not have much information about them in the fields of higher degrees. We will show how to get some of them in the certain family of cubic fields considering the Jacobi-Perron algorithm, which generates one type of multidimensional continued fractions. This is joint work with Vítězslav Kala.

Partition relations

Contributed talk

Mgr. Dávid Uhrik

E-mail: david.uhrik@gmail.com *Study branch:* 4M1 – Algebra, number theory, and mathematical logic *Year of study:* 1. *Supervisor:* RNDr. David Chodounský, Ph.D.

Abstract

The study of infinite partition relations arguably started in 1930 when Ramsey proved that given a graph on countably many vertices it is possible to find a countable subgraph such that either it or its complement is complete, symbolically $\omega \rightarrow (\omega, \omega)^2$. A straightforward generalization of this result to uncountable cardinalities is not possible. The method of forcing is often applied to get consistency results where questions cannot be settled in ZFC only.

Recollements associated to cotorsion pairs over upper triangular matrix rings

Contributed talk

M.Sc. Rongmin Zhu

E-mail: zhu@karlin.mff.cuni.cz *Study branch:* 4M1 – Algebra, number theory, and mathematical logic *Year of study:* 3. *Supervisor:* prof. RNDr. Jan Trlifaj, CSc., DSc.

Abstract

Let *A*, *B* be two rings, *M* an *A*-*B*-bimodule and $T = \begin{pmatrix} A & M \\ 0 & B \end{pmatrix}$. Given two complete hereditary cotorsion pairs $(\mathscr{A}_A, \mathscr{B}_A)$ and $(\mathscr{A}_B, \mathscr{B}_B)$ in *A*-Mod and *B*-Mod respectively. We define two cotorsion pairs $(\Phi(\mathscr{A}), \operatorname{Rep}(\mathscr{B}))$ and $(\operatorname{Rep}(\mathscr{A}), \Psi(\mathscr{B}))$ in *T*-Mod and show that both of these cotorsion pairs are complete and hereditary. Given two cofibrantly generated model structures \mathscr{M}_A and \mathscr{M}_B on *A*-Mod and *B*-Mod respectively. Using the result above, we investigate when there exist a cofibrantly generated model structure \mathscr{M}_T on *T*-Mod and a recollement of Ho (\mathscr{M}_T) relative to Ho (\mathscr{M}_A) and Ho (\mathscr{M}_B) . Finally, some applications are given in Gorenstein homological algebra.

Simplicial bundles and their application

Contributed talk

Mgr. Jiří Nárožný

E-mail: narozny@karlin.mff.cuni.cz

Study branch: 4M2 – Geometry and topology, global analysis and general structures *Year of study:* 2.

Supervisor: doc. Ing. Branislav Jurčo, CSc., DSc.

Abstract

One of the most promising physical theories which we have today is the string theory. In spite of its undoubted success, there are still many mathematical mysteries above it. One of those key concepts, nowadays still poorly understood, are higher gauge fields, also known as background gauge fields. From the mathematical point of view, any higher gauge field is a connection naturally living on generalized principal bundle, which we call the higher principal bundle. These bundles are defined in the language of higher category theory, modern mathematical tool which found its application not only in higher geometry I will discuss. The biggest disadvantage of the higher category theory is its complexity and it is practically impossible to compute anything in such playground, however. This is why the theory of simplicial sets was developed. More specifically we can say that simplicial sets are well behaved combinatorially flavoured replacements for higher categories which retain most of relevant information from each higher category. In this simplicial jargon, we can relatively directly formulate notions like simplicial manifolds as well as simplicial principal bundles.

In this talk I will expose some ideas and techniques from the category theory and ordinary differential geometry with the aim to generalize the well known construction of ordinary principal bundles to simplicial principal bundles. If time permits, I will show the methods how could be simplicial bundles furnished with higher connections, because this is the point where the true string theory begins.

The trouble with injectivity

Contributed talk

Mgr. Ondřej Bouchala

E-mail: ondrej.bouchala@gmail.com *Study branch:* 4M3 – Mathematical analysis *Year of study:* 1. *Supervisor:* prof. RNDr. Stanislav Hencl, Ph.D., DSc.

Abstract

Let us consider the class of homeomorphisms from $\Omega \subseteq \mathbb{R}^n$ to $\Omega' \subseteq \mathbb{R}^n$. There are many problems, where it is reasonable to consider solutions in this class. And even more reasonable is to look for solution in the closure of this class.

Therefore the functions we are interested in are a weak limits of homeomorphisms in $W^{1,p}(\Omega)$. Specifically we would like to know, if this functions must be injective. Of course the problem is also to define what does "injective" mean, because we have only functions that are defined almost everywhere. Therefore we need to chose good representative, and we need to define injectivity "almost everywhere", where almost everywhere could be in the domain or in the image.

Reduction principles and their applications

Contributed talk

RNDr. Zdeněk Mihula

E-mail: mihulaz@karlin.mff.cuni.cz *Study branch:* 4M3 – Mathematical analysis *Year of study:* 2. *Supervisor:* prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

We introduce the idea of so-called *reduction principles* in function spaces. We outline how those principles can be used for reducing complicated questions (e.g. Sobolev-type embeddings, boundedness of operators, etc.) to much simpler and more manageable questions.

Banach-Stone type theorems for subspaces of continuous functions

Contributed talk

Mgr. Jakub Rondoš

E-mail: jakub.rondos@gmail.com *Study branch:* 4M3 – Mathematical analysis *Year of study:* 2. *Supervisor:* prof. RNDr. Jiří Spurný, Ph.D., DSc.

Abstract

The classical Banach-Stone theorem asserts that the topological structure of a compact Hausdorff space is completely determined by the linearmetric structure of its space of real-valued continuous functions. More precisely, given a pair of compact Hausdorff spaces K and L, they are homeomorphic provided that the spaces $\mathscr{C}(K)$ and $\mathscr{C}(L)$ are isometrically isomorphic. One might ask what can be said if the spaces of continuous functions are replaced by closed subspaces $\mathscr{H}_1 \subseteq \mathscr{C}(K)$ and $\mathscr{H}_2 \subseteq \mathscr{C}(L)$. We present joint results of me and my supervisor Jiří Spurný concerning this question. Next we speak about related problems for vector-valued functions. Here the situation depends heavily on geometrical properties of the target space.

Mathematical analysis of models of fluids, solids, and their mutual interaction

Contributed talk

M.Sc. Jan Scherz

E-mail: jan.scherz@stud-mail.uni-wuerzburg.de *Study branch:* 4M3 – Mathematical analysis *Year of study:* 1. *Supervisor:* Mgr. Barbora Benešová, Ph.D.

Abstract

We consider a model of solid bodies moving in a compressible fluid, described by the Navier-Stokes equations. Approximating this system by a model containing no solid objects, we give an insight into the proof of the existence of a variational solution. Finally, we consider further physical properties which may be added to the model.

Monotone numerical schemes for convection-dominated problems Contributed talk

M.Sc. Shahin Heydari

E-mail: sh1990.heydari@gmail.com *Study branch:* 4M6 – Scientific computing *Year of study:* 1. *Supervisor:* doc. Mgr. Petr Knobloch, Dr., DSc.

Abstract

The numerical stability of convection-diffusion equations is mostly due to presence of the diffusion term. Then, when convection dominates diffusion it deprives the Galerkin FEM of the best approximation properties. Hence, appearing instabilities in numerical results is quite natural. These instabilities results in the presence of large overshoot and undershoots, which are sign of a violation of discrete maximum principle (DMP). The lack of robustness can be rectified by applying stabilized discretization. Despite all the features that stabilized discretization have, there are still non-negligible spurious oscillations in a vicinity of sharp layer. Due to the demand for the property of being free of oscillations, nonlinear discretization became of interest. One particular nonlinear discretization, designed to satisfy the DMP is the one known as algebraic flux correction (AFC) method.

The aim of this talk is to give an overview of AFC method and next challenge or open problem in this direction.

Numerical solution of traffic flow models

Contributed talk

Mgr. Lukáš Vacek

E-mail: lukas.vacek6@gmail.com *Study branch:* 4M6 – Scientific computing *Year of study:* 1. *Supervisor:* doc. RNDr. Václav Kučera, Ph.D.

Abstract

Modelling of traffic flows will have an important role in the future. With a rising number of cars on the roads, we must optimize the traffic situation. That is the reason we started to study traffic flows. It is important to have working models which can help us to improve traffic flow. We can model real traffic situations and optimize e.g. the timing of traffic lights or local changes in the speed limit. The benefits of modeling and optimization of traffic flows are both ecological and economical.

In (Vacek, 2018) we describe the numerical solution of traffic flows on networks. There are two fundamental approaches: microscopic and macroscopic. The microscopic approach follows every single car. The macroscopic approach consider the traffic to be a continuum which flows through the roads.

We solve especially the macroscopic models. Using these models, it is possible to make simulations on big networks with a lot of cars. These models are described by partial differential equations. For the numerical solution of our models, we use the discontinuous Galerkin method in space and a multistep method in time. We introduced limiters which prevent spurious oscillations in the numerical solution.

We present several numerical results. We compare different traffic flow approaches such as Greenshields model, Greenberg model and Underwood model. We present the behavior at junctions. Finally, we can compare our result with real data on highways.

The study was supported by the Charles University, project GA UK No. 1114119.

Using tablets in mathematics classes

Contributed talk

Mgr. Yulianna Tolkunova

E-mail: katapulta.yul@gmail.com *Study branch:* 4M8 – General questions of mathematics and computer science *Year of study:* 3. *Supervisor:* doc. RNDr. Jarmila Robová, CSc.

Abstract

Presented study deals with students' opinion on involving tablets in mathematics lessons and shows how the opinion on using tablets is changing after a one-year implementation. The study held over one school year and explored ten classes (over 260 students) with two different teachers in two different charter schools in USA. The study employed qualitative methods. In the session I will highlight findings from input and output questionnaires, observations and interviews, which took place in order to find out about using tablets in mathematics lessons. From the study you can learn whether students like or dislike using tablets in mathematics classes and what aspects can influence their attitude.

Peer instruction in elementary school mathematics

Contributed talk

Mgr. Tomáš Zadražil

E-mail: tomas.zadrazil@gmail.com *Study branch:* 4M8 – General questions of mathematics and computer science *Year of study:* 2. *Supervisor:* doc. RNDr. Jarmila Robová, CSc.

Abstract

Peer instruction is an active learning method which was primarily invented and popularised by Eric Mazur for the needs of university level physics. Ever since Peer instruction have expanded to every science discipline on all levels of education.

The effectiveness of this method stands primarily on the group discussion that was raised by the conceptual question of the so-called ConcepTest.

There were quite a lot of studies of Peer instruction but mostly engaged on its application in physics.

In this contribution we will introduce Peer instruction. We will see how to implement this strategy in teaching of elementary school mathematics and supporting it by a few examples of elementary mathematics ConcepTests. There will be presented design and current research status which is aimed at implementation of Peer instruction. Especially these two questions should be answered:

- 1. Is it possible to fulfill premises of Peer instruction in elementary school level?
- 2. Are there similar outcomes which were promised by foreign studies of this method in physics or in the university level teaching?

Multivariate associations measures

Contributed talk

Mgr. Vojtěch Kika

E-mail: vojtech.kika@kuleuven.be *Study branch:* 4M9 – Probability and statistics, econometry and financial math. *Year of study:* 2. *Supervisor:* doc. Ing. Marek Omelka, Ph.D.

Abstract

Coefficients like Kendall's tau or Spearman's rho, used to measure (strength of) an association between two random variables, were thoroughly studied and described in the middle of 20th century. Requirements on bivariate association measures are well-known. However, generalization of such measures into higher dimensions is not very straightforward and brings discussion on the desirable properties. In addition, bivariate measures can be often generalized in multiple manners. We discuss methods of generalization, their theoretical properties, strengths and weaknesses. In particular, behavior of association measures when dimension grows to infinity is of interest. Moreover, examples illustrate the performance and use of such measures.

Copulas in extreme-value theory

Contributed talk

Mgr. Vít Kubelka

E-mail: kubelka@karlin.mff.cuni.cz *Study branch:* 4M9 – Probability and statistics, econometry and financial math. *Year of study:* 3. *Supervisor:* prof. RNDr. Bohdan Maslowski, DrSc.

Abstract

Copulas model dependence in multivariate random distributions regardless of the marginal distributions. We say that a multivariate distribution with a distribution function F is max-infinitely divisible (max-id) if and only if F^t is a distribution function for all t > 0. Max-id distributions play a key role in extreme-value theory, therefore, it is important to study copulas which create a max-id distribution. Such copulas are called max-id.

First, we will briefly recall the theory of copulas and extreme-value theory. Then, using a Poisson random measure we will construct a new class of max-id copulas. We will find extremal attractors of these new copulas and we will state a condition under which these new copulas are extreme-value copulas themselves.

Classification based on longitudinal data of a mixed type *Contributed talk*

Mgr. Jan Vávra

E-mail: vavraj@karlin.mff.cuni.cz *Study branch:* 4M9 – Probability and statistics, econometry and financial math. *Year of study:* 1. *Supervisor:* doc. RNDr. Arnošt Komárek, Ph.D.

Abstract

In different types of studies an unobserved heterogeneity within the data is suspected. To reveal homogeneous groups of similar units and construct a rule on how to classify future units a suitable method of statistical classification can be used. However, commonly used methods of classification usually cannot be directly applied in the case of longitudinally measured multivariate outcomes, especially if those can be both continuous and discrete, i.e., longitudinal data of a mixed type. A modern concept of classification called *Model Based Clustering* (MBC) that becomes very popular nowadays, is adopted to this situation. This approach requires building a multivariate probabilistic model for mixed type longitudinal data. Therefore, relevant univariate models for longitudinal numeric, binary and ordinal variables are reviewed and a method of joining these models into the desired complex multivariate model is proposed. Suggested model is of a complex hierarchical structure that requires Bayesian approach and MCMC methodology to infer on the model parameters. EU-SILC dataset (The European Union Statistics on Income and Living Conditions) serves not only as a motivation example but also as an illustration of application of developed methods.

Thermomechanical interaction of ice shells and subsurface oceans *Contributed talk*

Mgr. Jiří Malík

E-mail: malik@karlin.mff.cuni.cz *Study branch:* 4F11 – Mathematical and computer modeling *Year of study:* 1. *Supervisor:* RNDr. Ondřej Souček, Ph.D.

Abstract

Thermomechanical interaction between ice shell and subsurface ocean in the context of planetary sciences can give some information about the early evolution of certain objects in our Solar system. In particular objects that are interesting from the point of view of astrobiology. Questions related to the existence of life and their relevance to mathematical modeling of phase change problems will be addressed in this talk. Mathematical formulation of a thermomechanical evolution of two-phase continuum in a two-dimensional domain with free surface contain implementational and computational difficulties. Those aspects of the formulation will be discussed in the rest of the talk.

Natural configuration, GENERIC and anisotropic viscoelasticity Contributed talk

Mgr. Petr Pelech

E-mail: pelech@karlin.mff.cuni.cz *Study branch:* 4F11 – Mathematical and computer modeling *Year of study:* 3. *Supervisor:* doc. RNDr. Martin Kružík, Ph.D., DSc.

Abstract

Non-equilibrium thermodynamics is a lively evolving scientific field with many competitive theories. Our goal is to establish a connection between two of them: GENERIC and the framework of natural configuration. We aim to formulate an anisotropic Maxwell model equipped with both conformation (or left Cauchy-Green) tensor and an advected vector field describing the immersed fibers. Doing so in both of these frameworks in parallel should help to translate ideas from one theory to the other. Preliminary results concerning the formulation within GENERIC will be presented. It is a joint work with Josef Málek, Michal Pavelka and Karel Tůma (all from Charles University).

Line-items and table understanding in structured documents Poster Mgr. Martin Holecek

E-mail: mholecek91@volny.cz *Study branch:* 4M6 – Scientific computing *Year of study:* 3. *Supervisor:* prof. Ing. František Maršík, DrSc.

Modeling of porous metal oxide layer growth Poster

Mgr. Michal Bathory

E-mail: bathory@karlin.mff.cuni.cz *Study branch:* 4F11 – Mathematical and computer modeling *Year of study:* 3. *Supervisor:* RNDr. Miroslav Bulíček, Ph.D.

Gradient polyconvexity and rate-independent evolution of shape memory alloys

Poster

Mgr. Petr Pelech

E-mail: pelech@karlin.mff.cuni.cz *Study branch:* 4F11 – Mathematical and computer modeling *Year of study:* 3. *Supervisor:* doc. RNDr. Martin Kružík, Ph.D., DSc.