

Předmět NM AFODG

19.5.2020

(9. diskuse)

OPAKOVÁNÍ

$$P = e^{-x^2} \int_{-\infty}^{\infty} e^{-x^2} (P(x))^2 dx$$

Věta $L_p^2(a, b)$, $-\infty \leq a < b \leq \infty$, P máha, taková,

\tilde{v} $P(x) \in L_p^2(a, b)$ & P polynom. Bud $\{\varphi_m\}$ soubor reálných DG polynomů v L_p^2 , tak $\varphi_m = m$, $m = 0, 1, 2, \dots$

Položme $H_m \exists A_m, B_m, C_m \in \mathbb{R}$, \tilde{v}

$$(*) \quad x\varphi_m = A_m \varphi_{m+1} + C_m \varphi_m + B_m \varphi_{m-1}$$

Diskuse: • $A_m \neq 0$ (protože $\text{rk}(x\varphi_m) \approx m+1$)

• vynásob $(*)$ $(\cdot, \varphi_{m+1})_{2,p}$:

$$\rightarrow (x\varphi_m, \varphi_{m+1})_{2,p} = A_m \|\varphi_{m+1}\|_{2,p}^2 + 0 + 0$$

$m \geq 1$ • vynásob $(*)$ $(\cdot, \varphi_{m-1})_{2,p}$

$$\tilde{v} (x\varphi_m, \varphi_{m-1})_{2,p} = 0 + 0 + B_m \|\varphi_{m-1}\|_{2,p}^2$$

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$$\left\{ \begin{array}{l} (\chi \varphi_{m-1}, \varphi_m)_{2,p} = A_{m-1} \|\varphi_m\|_{2,p}^2 \\ \int p(x) \chi \varphi_{m-1} \cdot \varphi_m \\ \quad " \\ \int p(x) \chi \varphi \cdot \varphi_{m-1} \\ \quad " \\ (\chi \varphi_m, \varphi_{m-1})_{2,p} = B_m \|\varphi_{m-1}\|_{2,p}^2 \end{array} \right.$$

$$A_{m-1} \|\varphi_m\|_{2,p}^2 = B_m \|\varphi_{m-1}\|_{2,p}^2$$

$\frac{\#}{\#} \Rightarrow \frac{0}{0}$

$$\|\varphi_m\|_{2,p}^2 = \frac{B_m}{A_{m-1}} \|\varphi_{m-1}\|_{2,p}^2$$

$m \geq 1$

6.3 Gaussova reakčná maticová rovnica a ortog. sest. polynomov

Méting GRR:

$$xy'' + (\lambda + 1 - x)y' - \alpha y = 0 \quad x \neq 0$$

$$x \in (-\infty, 0) \cup (0, \infty)$$

$$\alpha, \lambda \in \mathbb{C}$$

$$\lambda \neq -1, -2, -3, \dots$$

$$\int_p^2 ?? \text{ jakej } p?$$

(prvok v řadě málo pravděpodobný)

Knoten:

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• Freiheit GRR

$$Ty = \lambda \rho y \quad (?)$$

↔ Ramrodij-Armen?

$$Ty = -(py')' \quad ?$$

Hedâne p, ρ , alg GRR
a par- λ

$$xy'' + (\Delta + 1 - x)y' - \alpha y = 0$$

||

$$-(py')' = \lambda \rho y$$

$x \neq 0$

$$-p'y' - py'' = \lambda \rho y$$

Priédyn.

$: (-p) \neq 0$

$$y'' + \frac{p'}{p} y = -\lambda \frac{\rho}{p} y$$

$$y'' + \left(\frac{\Delta+1}{x} - 1 \right) y = \frac{\alpha}{x} y$$

$$\frac{p'}{p} = \frac{\Delta+1}{x} - 1 \quad \& \quad \lambda = -\alpha \quad \& \quad \frac{\rho}{p} = \frac{1}{x}$$

$$(\ln|p'|)' = (\Delta+1)(\ln|x|)' - 1$$

$$|p'| = |x|^{\Delta+1} x^{-x}$$

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$$\boxed{x > 0} : \boxed{p(x) = x^{\alpha+1} e^{-x}}$$

$$\lambda = -\alpha$$

$$P = \frac{P}{x} = x^\alpha e^{-x}$$

$$\boxed{\int_0^x P(t) dt}$$

$$\Rightarrow \text{Pracížime na } \boxed{x^\alpha e^{-x} (0, +\infty)}$$

Vlastnosti p: $p > 0$ ✓

$$p \in L^1(0, \infty) \quad \int_0^\infty |x^\alpha e^{-x}| < \infty$$

$$\text{Re } \alpha > -1$$

GRR \Leftrightarrow $\boxed{x > 0} \quad \boxed{-((x^{\alpha+1} e^{-x})' y')' = (-\alpha) x^\alpha e^{-x} y''}$

$(0, \infty)$ ✓ domenadž. tvary

P

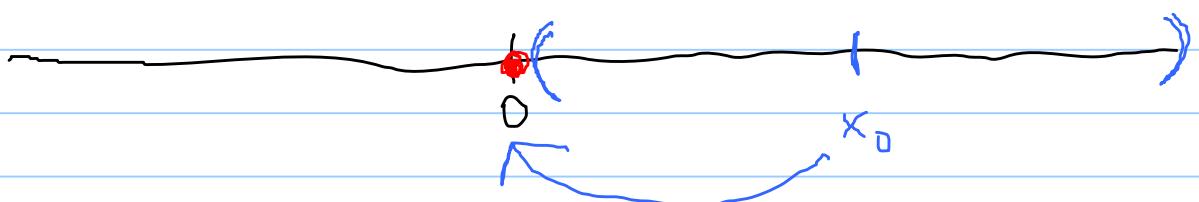
② Žesúme možnosť niečo GRR

$$\rightarrow xy'' + (\alpha+1-x)y' - \alpha y = 0$$

$(-\infty, 0) \cup (0, \infty)$ Řešení v lewo vpravo

$$y = \sum_{m=0}^{\infty} c_m (x - x_0)^m$$

SLEPIT!



Hledáme řešení ve formě $y = \sum_{n=0}^{\infty} c_n x^n$

$$\sum_{n=2}^{\infty} c_n n(n-1)x^{n-1} \cdot x$$

$$+ (\alpha + 1) \sum_{n=1}^{\infty} c_n n x^{n-1} - \sum_{n=1}^{\infty} c_n n x^n - \alpha \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=1}^{\infty} c_{n+\alpha} (n+\alpha)n x^n + (\alpha + 1) \sum_{n=0}^{\infty} c_{n+1} (n+\alpha+1) x^n - \sum_{n=1}^{\infty} c_n n x^n \\ - \sum_{n=0}^{\infty} \alpha c_n x^n = 0$$

$$m \quad x^0 : \quad (\alpha + 1) c_1 = \alpha c_0 \Rightarrow c_1 = c_0 \frac{\alpha}{\alpha + 1}$$

$\alpha \neq -1$

$$m \geq 1 \quad x^m : \quad c_{m+1} (m+1) [m + \alpha + 1] = c_m (m + \alpha)$$

$$m = 0, 1, 2, \dots \quad \boxed{c_{m+1} = c_m \frac{m + \alpha}{(m + 1)(m + \alpha + 1)}}$$

$\alpha \neq -2, -3, -4, \dots$

c_0 volit

BÚNO

$$\boxed{c_0 = 1}$$

Konečně nás iada? Kde? $R = \frac{1}{\lim_{m \rightarrow \infty} \frac{c_{m+1}}{c_m}}$

$R = +\infty$

\square

INTERMEZZO : HYPERGEOMETRISCHE RÄDY

Def: HG-Rada je močinska rada $\sum_{m=0}^{\infty} c_m x^m$,

glej \Rightarrow koef. definij:

a) \exists poljaz $P, Q \in \mathbb{R}[x]$ - u nevajš močig
vred ≥ 1 , $\Delta P = p \geq 0$, $\Delta Q = q \geq 0$,

Q nemá krajnem cisl IN U {0}

b)

$$\left[\begin{array}{l} \frac{c_{m+1}}{c_m} = \frac{P(m)}{Q(m)} \cdot \frac{1}{m+1} \\ c_0 = 1 \end{array} \right] \quad m = 0, 1, 2, 3.$$

hist. obrazec

Prim: $P(m) = Q(m)(m+1) \Rightarrow \frac{c_{m+1}}{c_m} = 1$

pro rada $\sum c_m x^m \quad \left| \frac{c_{m+1} x^{m+1}}{c_m x^m} \right| = |x|$

geom.-r. s krac x

Zvezek P, Q na konvergencibele

$$\rightarrow \left| \frac{c_{m+1}}{c_m} \right| = \frac{(a_1+m)(a_2+m) \dots (a_p+m)}{(b_1+m)(b_2+m) \dots (b_q+m) \cdot (m+1)}$$

Tato situaci je nadejno najden:

$$\sum_{m=0}^{\infty} c_m x^m = {}_p F_q [a_1, \dots, a_p; b_1, \dots, b_q] (x)$$

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Pom.: $\int_0^x \left[\begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} \right] (x) \dots$

Σ koeficientů nazíváme hodnotou řídkého řídého řadu $\sum c_n x^n$
konvergencie:

(1) $\boxed{p < q+1} \Rightarrow \left| \frac{c_{n+1}}{c_n} \right| \rightarrow 0 \Rightarrow R = +\infty$

$$\sum c_n x^n \in C^\infty(-\infty, \infty)$$

(2) $\boxed{p = q+1} \Rightarrow \left| \frac{c_{n+1}}{c_n} \right| \rightarrow 1 \Rightarrow R = 1$ (v $\mathcal{A}(C)$)

$$\sum c_n x^n \in C^0(-1, 1)$$

(3) $\boxed{p > q+1} \Rightarrow \left| \frac{c_{n+1}}{c_n} \right| \rightarrow \infty \Rightarrow R = 0$ X (v $\mathcal{A}(k, 0)$)

Cíl: 2 rekurrence pro c_{n+1} odvozit typ. funk.

↓
žeží 1 symbol „n dřívých drah“

POCHHAMMERŮV SYMBOL

$$a \in \mathbb{C} \quad (a)_0 = 1$$

$$(a)_m = a(a+1) \cdots (a+m-1)$$

m členů

rising factor
"real"

Plán:

$$(1)_m = m! ; (a)_m = \frac{\Gamma(a+m)}{\Gamma(a)}$$

$$(2)_4 = 2 \cdot 3 \cdot 4 \cdot 5$$

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$$\frac{c_{m+1}}{c_m} = \frac{(a_1+m)(a_2+m) \dots (a_p+m)}{(b_1+m)(b_2+m) \dots (b_q+m) \cdot (m+1)}$$

$$c_m = \frac{(a_1+m-1)(a_2+m-1) \dots (a_p+m-1)}{(b_1+m-1)(b_2+m-1) \dots (b_q+m-1)} \cdot \frac{1}{m} \cdot c_{m-1} =$$

$$= \frac{[(a_1+m-1)(a_1+m-2)] \{ (a_2+m-1)(a_2+m-2) \} \dots \frac{1}{m(m-1)} c_{m-2}}{b^{11}}$$

$$= \frac{(a_1)_m \dots (a_p)_m}{(b_1)_m \dots (b_q)_m} \cdot \frac{1}{m!} c_0 = \frac{\prod_{j=1}^p (a_j)_m}{\prod_{j=1}^q (b_j)_m} \cdot \frac{1}{m!}$$

$$F_g[a_1, \dots, a_p; b_1, \dots, b_q](x) = \sum_{m=0}^{\infty} \frac{\prod_{j=1}^p (a_j)_m}{\prod_{j=1}^q (b_j)_m} \cdot \frac{x^m}{m!}$$

$a_i, b_j \in \mathbb{R}$

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T.

$${}_0F_0 [;](x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

$\textcircled{1}$

$${}_0F_1 \left[; \frac{1}{2} \right] \left(-\frac{x^2}{4} \right) = \sum_{m=0}^{\infty} \frac{1}{\left(\frac{1}{2} \right)_m} \cdot \frac{1}{m!} \left(-\frac{x^2}{4} \right)^m = \\ = \sum_{m=0}^{\infty} (-1)^m \frac{x^{2m}}{(2m)!} = \cos x$$

$$\frac{2x}{\sqrt{\pi}} {}_1F_1 \left[\frac{1}{2}; \frac{3}{2} \right] (-x^2) = \frac{1}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

pro $x \in \mathbb{R}$

$p=1 \quad q=1$

$p < q+1$

$\mathcal{X}(C)$

$\operatorname{Ernf}(x)$

KONEC INTERMEZZA

KONEC PREDN. 19.5.