

INTRODUCTION TO GROUP THEORY – NMAG337

EXAM – JANUARY 10 2023

**Problem 1.** *Sylow's theorem – formulate and sketch proofs, include supporting lemmas, possibly show some applications.*

**Problem 2.** *Prove that the alternating group  $A_n$  is simple for all  $n \geq 5$ .*

**Problem 3.** *Prove that the dihedral group  $D_n$  (of all symmetries of a regular  $n$ -gon) is a semidirect of a cyclic group  $\mathbb{Z}_n$  by  $\mathbb{Z}_2$ .*