Riemannian geometry II
Exercises for zápočet

In the following exercises you may use any theorem stated in the course.

(1) Compute the spectrum of the Laplace-Beltrami operator on the real projective space $\mathbb{RP}^n$ of dimension n with respect to the standard $SO(n+1)$-invariant Riemannian metric (for which the canonical projection from the n-dimensional sphere S^n to $\mathbb{RP}^n$ is a local isometry).

(2) Show that any harmonic differential 1-form (with respect to the round metric) on the 2-dimensional sphere S^2 is zero. (Hint: work in the coordinates determined by stereographic projection.)