

4.1

GEOMETRIE 1

$$c(t) = \begin{pmatrix} t \\ t^3 \end{pmatrix} \quad c'(t) = \begin{pmatrix} 1 \\ 3t^2 \end{pmatrix} \quad c''(t) = \begin{pmatrix} 0 \\ 6t \end{pmatrix}$$

$$c(1) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad c'(1) = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad c''(1) = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

$$\mathcal{K}_2(1) = \frac{\begin{vmatrix} 1 & 0 \\ 3 & 6 \end{vmatrix}}{\left\| \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\|^3} = \frac{6}{10\sqrt{10}} = \frac{3}{5\sqrt{10}}$$

$$\vec{t}(1) = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \vec{n}_*(1) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \frac{1}{\sqrt{10}} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

TANGENTE: $\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\rangle$
 $-3x + y + 2 = 0$

NORMALE: $\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right\rangle$
 $x + 3y - 4 = 0$

$$R(1) = \frac{1}{\mathcal{K}_2(1)} = \frac{5}{3}\sqrt{10}$$

$$S(1) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + R(1) \vec{n}_*(1) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{5}{3}\sqrt{10} \frac{1}{\sqrt{10}} \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ \frac{8}{3} \end{pmatrix}$$

$$\tilde{c}(s) = (s^3, s^9)^T \quad t=1 = s^3 \Rightarrow s=1$$

$$\tilde{c}'(s) = \begin{pmatrix} 3s^2 \\ 9s^8 \end{pmatrix} \quad \tilde{c}''(s) = \begin{pmatrix} 6s \\ 72s^7 \end{pmatrix}$$

$$\mathcal{K}_2(1) = \frac{\begin{vmatrix} 3 & 6 \\ 9 & 72 \end{vmatrix}}{\left\| \begin{pmatrix} 3 \\ 9 \end{pmatrix} \right\|^3} = \frac{162}{3^3 \cdot 10 \cdot \sqrt{10}} = \frac{3}{5\sqrt{10}}$$