

$$\mathbb{R}^2 \subset \mathbb{P}^2 :$$

$$3x^2 + 2xy + 3y^2 + 6x - 2y - 5 = 0$$

$$(x_1, x_2, x_3) \begin{pmatrix} 3 & 1 & 3 \\ 1 & 3 & -1 \\ 3 & -1 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

B

+ - 0
signature $B = (2, 1, 0)$

$$\begin{matrix} * \\ + \\ - \end{matrix}$$

$$\Delta_0 = 1 \quad \Delta_1 = 3 \quad \Delta_2 = 8 \quad \Delta_3 = -76$$

$$\det B = -76 \Rightarrow \text{regulär}$$

Affin typ $x_3 = 0$



$$(0, 0, 1)^*$$

$$x_3 = 0$$

P_{∞}

$$3x_1^2 + 2x_1x_2 + 3x_2^2 = 0$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

signature

$$(2, 0, 0)$$

pos. def. \Rightarrow ellipse

$$D = 4 - 4 \cdot 9 < 0$$

$$\Rightarrow 0 \text{ reellen}$$

$$\Rightarrow \text{ellipse!}$$

Stütz

pol P_{∞}

$$S = B^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^* = \begin{pmatrix} -\frac{5}{4} \\ \frac{3}{4} \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} -\frac{5}{4} & \frac{3}{4} \end{bmatrix}$$

$$B \cdot S = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^*$$

P_{∞}

\uparrow pol

\uparrow pol

$$X = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

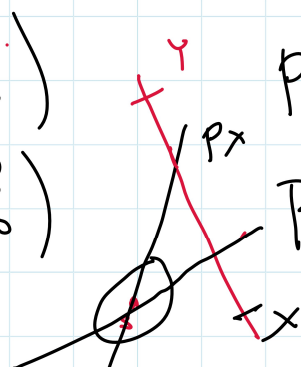
$$Y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$P_X = B \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}^*$$

$$P_Y = B \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}^*$$

$$S = P_X \cap P_Y$$

P_X



Hlavní
směry

$X, Y \leftarrow$ polární souřadnice a kořeny
 $X, Y \in \mathbb{R}^3$

$$\begin{matrix} Y & & X \\ (-b, a, 0) & \cdot & \begin{pmatrix} 3 & 1 & 3 \\ 1 & 3 & -1 \\ 3 & -1 & -5 \end{pmatrix} & \cdot & \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} & = & 0 \end{matrix}$$

$\underbrace{\hspace{10em}}_B$

$$\begin{aligned} (-b, a, 0) \cdot \begin{pmatrix} 3a + b \\ a + 3b \\ a \end{pmatrix} &= -3ab - b^2 + a^2 + 3ab = 0 \\ &= a^2 - b^2 = 0 \\ &= (a+b)(a-b) = 0 \end{aligned}$$

$$\begin{aligned} X &= (1, -1, 0) \\ Y &= (1, 1, 0) \end{aligned}$$

