$$\left\{ \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \;\middle|\; \underbrace{(x_1 \dots x_n \mid 1)}_{\vec{x}^T} \left(\begin{array}{c|c} A & b \\ \hline b^T & c \end{array}\right) \begin{pmatrix} x_1 \\ \vdots \\ x_n \\ 1 \end{pmatrix} = 0 \right\}$$

$A = A^T$

$$\boxed{\vec{x}^T A \vec{x} + 2\vec{b}^T \vec{x} + c = 0}$$

sign A, sign $\begin{pmatrix} A & b \\ \vec{b}^T & c \end{pmatrix}$ nemění typ kvadriky

afinní

$n = 2, 3$, $(\dots)$

$$\left(\begin{array}{c|c} A & \vec{b} \\ \hline \vec{b}^T & c \end{array}\right)$$

regulární

$$\left(\begin{array}{c|c} D & \vec{0} \\ \hline \vec{0}^T & c' \end{array}\right) \dots D \text{ reg. } \quad d_1 x_1^2 + \dots + d_n x_n^2 + c' = 0$$

$$\left(\begin{array}{cc|c} \tilde{D} & 0 & 0 \\ 0 & 0 & 1 \\ \hline 0 \dots 0 & 1 & 0 \end{array}\right) \dots \tilde{D} \text{ reg } \quad \tilde{d}_1 x_1^2 + \dots + \tilde{d}_{n-1} x_{n-1}^2 + 2x_n = 0$$

sign A = sign D, resp. sign $(\tilde{D})$

Jak určit sign A?

1) Symetrické úpravy

$$R A R^T = \tilde{D} \quad \text{diag.}$$

sign se zachová!

2) Vlastní čísla

U je ortogonální

$$U A U^T = \tilde{D}$$

na diag. jsou vlastní čísla A

3) Sylvestrovo kritérium

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

A_1, A_2, A_3

znaménka v posloupnosti

$$\frac{1}{\det A_1}, \quad \frac{\det A_1}{\det A_2}, \quad \frac{\det A_2}{\det A_3}$$

jsou stejná jako znaménka na diag. $\tilde{D}$.

11.1

$$(x_1, x_2, x_3)\begin{pmatrix} -7 & -6 & 7 \\ -6 & 6 & -3 \\ 7 & -3 & 9 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$(1\ 5\ 1)\begin{pmatrix} -7 & -6 & 7 \\ -6 & 6 & -3 \\ 7 & -3 & 9 \end{pmatrix} = (-30, 21, 1)$$

$$\begin{pmatrix} 1 & 1 & 3 \\ -30 & 21 & 1 \end{pmatrix} \cdot 30 \sim \begin{pmatrix} 1 & 1 & 3 \\ 0 & 51 & 91 \end{pmatrix} \qquad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 62 \\ 91 \\ -51 \end{pmatrix}$$

11.2 a) Polára $[2,2]$ má rovnici b) Pól: $[a,b]$

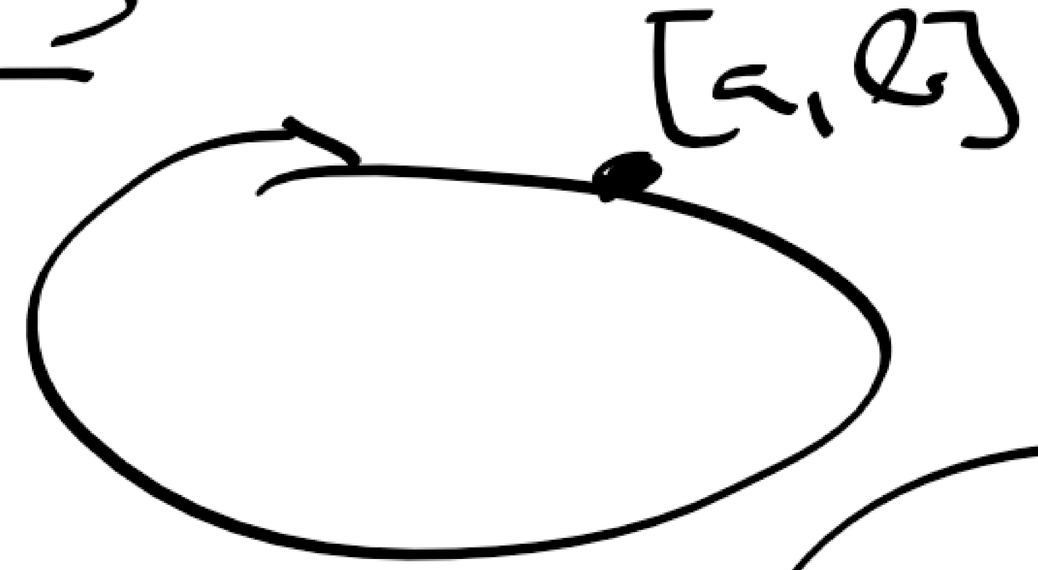
$$\underbrace{(2\ 2\ 1)\begin{pmatrix} 1 & -\frac{3}{2} & 0 \\ -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \qquad \underbrace{(a, b, 1)\begin{pmatrix} 1 & -\frac{3}{2} & 0 \\ -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} = k(1, 1, -2)$$

$$\boxed{-x-y+1} \;\; // \;\; \Leftarrow (-1\ \ -1\ \ 1) \qquad \left(a-\tfrac{3}{2}b, -\tfrac{3}{2}a+b, 1\right) \Rightarrow k = -\tfrac{1}{2}$$

c)
$$(1,2,0)\begin{pmatrix} 1 & -\frac{3}{2} & 0 \\ -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = 0 \Leftrightarrow -2x + \tfrac{1}{2}y = 0 \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\begin{aligned} 2a - 3b &= -1 \\ -3a + 2b &= 1 \end{aligned} \Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

11.3



$$\left\{ (a,b,1)\begin{pmatrix} 1 & \frac{1}{2} & 1 \\ \frac{1}{2} & 1 & \frac{3}{2} \\ 1 & \frac{3}{2} & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \right\}$$

- LEŽÍ NA KUŽELOSEČCE $\Leftrightarrow$

$$(a,b,1)\begin{pmatrix} 1 & \frac{1}{2} & 1 \\ \frac{1}{2} & 1 & \frac{3}{2} \\ 1 & \frac{3}{2} & -3 \end{pmatrix}\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} = 0$$

- TEČNA V BODĚ (= POLÁRA) MÁ NORMÁLU $(1,1)$

$$\Leftrightarrow (a,b,1)\begin{pmatrix} 1 \\ \frac{1}{2} \\ 1 \end{pmatrix} = (a,b,1)\begin{pmatrix} \frac{1}{2} \\ 1 \\ \frac{3}{2} \end{pmatrix}$$

$$a + \frac{1}{2}b + 1 = \frac{1}{2}a + b + \frac{3}{2} \Leftrightarrow \frac{1}{2}a = \frac{1}{2}b + \frac{1}{2}$$

$$a = b + 1$$

$$(b+1)^2 + (b+1)b + b^2 + 2(b+1) + 3b - 3 =$$

$$= b^2 + 2b + 1 + b^2 + b + b^2 + 2b + 2 + 3b - 3 = 3b^2 + 8b$$

$\Rightarrow [1,0]$ a $\left[-\frac{5}{3}, -\frac{8}{3}\right]$

$$(1,0,1)\begin{pmatrix} 1 & \frac{1}{2} & 1 \\ \frac{1}{2} & 1 & \frac{3}{2} \\ 1 & \frac{3}{2} & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \underline{2x + 2y - 2 = 0}$$

$$\left(-\frac{5}{3}, -\frac{8}{3}, 1\right) \text{———} \;''\; \text{———} = \underline{-2x - 2y - \frac{26}{3} = 0}$$