4.5.

$$\underbrace{x^4 - 2xy^3 + y^4}_{F(x,y)} = 0 \quad \text{v bodě } (1,1).$$

$$\nabla F\Big|_{(1,1)} = (F_x, F_y)\Big|_{(1,1)} = (4x^3 - 2y^3, -6xy^2 + 4y^3)\Big|_{(1,1)} =$$

$$= (2, -2).$$

Věta o impl. fcích: $\exists\ y = f(x)$ na okolí $x = 1$, přičemž f je hladká! a $\boxed{f(1) = 1}$.

Uvažujme tedy parametrizaci $c(t) = (t, f(t))$; $t \in (1-\varepsilon, 1+\varepsilon)$.

Pro ni platí

1) $t^4 - 2t f(t)^3 + f(t)^4 = 0 \quad \Big|\ \frac{\partial}{\partial t}$

2) $4t^3 - 2f(t)^3 - 6t f'(t) f(t)^2 + 4 f'(t) f(t)^3 = 0 \quad \Big|\ \frac{\partial}{\partial t}$

3) $12t^2 - 6 f'(t) f(t)^2 - 6 f'(t) f(t)^2 - 6t f''(t) \cdot f(t)^2 - 12t f'(t)^2 f(t)$
$+ 4 f''(t) \cdot f(t)^3 + 12 f'(t) \cdot f(t)^2 = 0$

Dosazením $t = 1$ do 2) dostáváme

$$4 - 2 - 6f'(1) + 4f'(1) = 0 \Rightarrow \boxed{f'(1) = 1}$$

a potom $t = 1$ do 3)

$$12 - 6 - 6 - 6f''(1) - 12 + 4f''(1) + 12 = 0$$

$$\Rightarrow f''(1) = 0.$$

tedy $c'(1) = (1,1)$ a $c''(1) = (0,0)$

$$\Rightarrow \underline{\underline{\kappa_z(1)}} = \frac{\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{\sqrt{2}^3} = 0$$