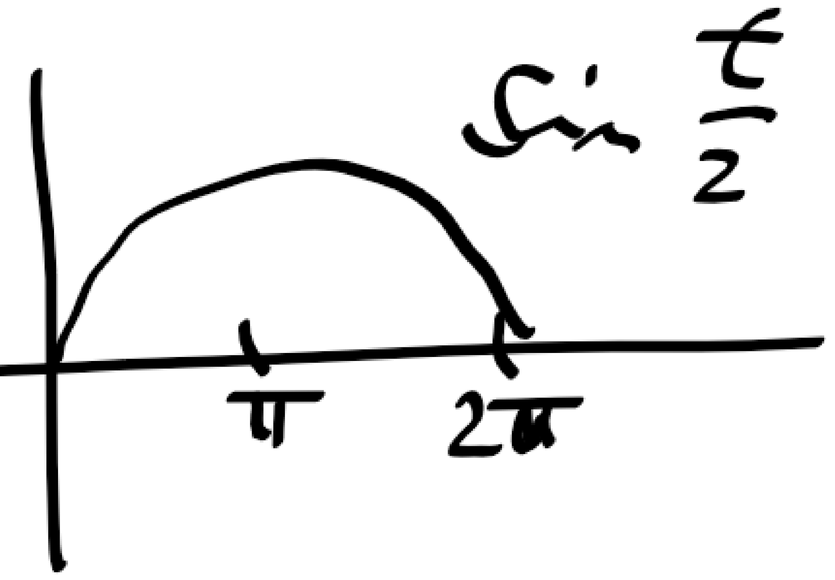
6.1

$$\int_0^{2\pi} \|c'(t)\|\, dt = 2\sqrt{2}\int_0^{2\pi} \sin\frac{t}{2}\, dt = 2\sqrt{2}\left[-2\cos\frac{t}{2}\right]_0^{2\pi}$$

$$= 4\sqrt{2}\left[-(-1)-(-(1))\right]$$

$$= 8\sqrt{2}$$



$$c'(t) = \left[1-\cos t,\ \sin t,\ -2\sin\frac{t}{2}\right]$$

$$\|c'(t)\| = \sqrt{(1-\cos t)^2 + (\sin t)^2 + 4\,\frac{1-\cos t}{2}} = \sqrt{1-2\cos t+\cos^2 t+\sin^2 t+2-2\cos t}$$

$$= \sqrt{4-4\cos t} = 2\sqrt{1-\cos t} = 2\sqrt{2}\,\left|\sin\frac{t}{2}\right|$$

6.2

$$c'(t) = \left[-3a\cos^2 t \sin t,\ 3a\sin^2 t\cos t\right]$$

$$\|c'(t)\| = 3a|\cos t \sin t|\sqrt{\cos^2 t + \sin^2 t} = \frac{3}{2}a|\sin 2t|$$

$$\int_0^{\frac{\pi}{2}} \frac{3}{2}a\sin 2t\,dt = \frac{3}{2}a\left[-\frac{1}{2}\cos 2t\right]_0^{\frac{\pi}{2}} = \frac{3}{2}a\left[\left(-\frac{1}{2}\right)(-1) - \left(-\frac{1}{2}\right)(+1)\right] =$$

$$= \frac{3}{2}a$$

6.3

$x = t$
$y = t^2$

$c'(t) = [1, 2t] \quad \|c'(t)\| = \sqrt{1+(2t)^2}$

$$\int_C |x|\, ds = \int_{-1}^{1} |t| \sqrt{1+4t^2}\, dt = 2\int_0^1 t\sqrt{1+4t^2}\, dt$$

$s = 1+4t^2$
$ds = 8t\, dt$

$$= \frac{1}{4}\int_1^5 \sqrt{s}\, ds = \frac{1}{4}\left[\frac{2}{3}s^{\frac{3}{2}}\right]_1^5 =$$

$$= \frac{1}{6}\left(5^{\frac{3}{2}} - 1\right)$$

6.4 $\quad \vec{c}'(t) = [a(1-\cos t),\ a\sin t] \qquad \vec{F}(x,y) = [a-y,\ x]$

$$\int_c (a-y)\,dx + x\,dy = \int_0^{2\pi} \vec{F}[\vec{c}(t)]\cdot\vec{c}\,'(t)\,dt =$$

$$= \int_0^{2\pi} \Big[\underbrace{(a\cos t)}_{\vec{F}_x(\vec{c}(t))}\underbrace{(a - a\cos t)}_{c'_x(t)} + \underbrace{a(t-\sin t)}_{\vec{F}_y(\vec{c}(t))}\,\underbrace{a\sin t}_{c'_y(t)}\Big]\,dt$$

$x = a(t-\sin t)$

$y = a - a\cos t$

$$= a^2\int_0^{2\pi}(\cos t - \cos^2 t + t\sin t - \sin^2 t)\,dt$$

$$= a^2\Big(\underbrace{\int_0^{2\pi}\cos t\,dt}_{0} - \underbrace{\int_0^{2\pi}1\,dt}_{2\pi} + \underbrace{\int_0^{2\pi}t\sin t\,dt}_{-2\pi}\Big) = \underline{\underline{-4\pi a^2}}$$

$$\int_0^{2\pi}\underset{u}{t}\,\underset{v'}{\sin t}\,dt = [\overset{u}{-t}\,\overset{v}{\cos t}]_0^{2\pi} - \int_0^{2\pi}1\cdot(-\cos t)\,dt$$

$$= [-2\pi\cdot 1 - 0] - 0 \overset{u'\,v}{=} -2\pi$$

6.5 $\vec{c}(t) = [\sin^3 t, \cos^3 t] \quad t \in [0, \frac{\pi}{2}]$

$\vec{c}\,'(t) = [3\sin^2 t \cos t, 3\cos^2 t(-\sin t)]$

$F_x(\vec{c}(t)) = -\frac{\cos^6 t}{\sin^5 t + \cos^5 t} \qquad F_y(\vec{c}(t)) = \frac{\sin^6 t}{\sin^5 t + \cos^5 t}$

$$\int_c \frac{-y^2}{x^{\frac{5}{3}} + y^{\frac{5}{3}}}\,dx + \frac{x^2}{x^{\frac{5}{3}} + y^{\frac{5}{3}}}\,dy = \int_0^{\frac{\pi}{2}} 3\,\frac{-\sin^2 t\cos^7 t - \cos^2 t \sin^7 t}{\sin^5 t + \cos^5 t}\,dt$$

$$= -3\int_0^{\frac{\pi}{2}} (-\sin^2 t \cos^2 t)\,\frac{\cos^5 t + \sin^5 t}{\sin^5 t + \cos^5 t}\,dt = -3\int_0^{\frac{\pi}{2}} \left(\frac{1}{2}\sin 2t\right)^2 dt$$

$$= -\frac{3}{8}\int_0^{\frac{\pi}{2}} (1 - \cos 4t)\,dt = -\frac{3\pi}{16}$$

6.6 $\quad c(t) = [a\cos t, b\sin t]$ $\quad t \in \langle 0, 2\pi)$ $\quad F_1 = x+y \quad F_2 = -(x-y)$

$$\int_c \vec{F}\, d\vec{X} = \int_{\mathrm{Int}\, c} -2\, dx\, dy = -2S \qquad \frac{\partial F_1}{\partial y} = 1 \qquad \frac{\partial F_2}{\partial x} = -1$$

S ... plocha elipsy $= ab\tilde{S}$, kde $\tilde{S}$ je plocha jednotkové kružnice, tj $\tilde{S} = \pi$

$$\int_c \vec{F}\, d\vec{X} = -2\pi ab.$$

Přímo: $\quad c'(t) = [-a\sin t, b\cos t]$

$$\int_0^{2\pi} \left[(a\cos t + b\sin t)(-a\sin t) - (a\cos t - b\sin t)(b\cos t) \right] dt$$

$$= \int_0^{2\pi} \left[(-a^2)\cos t \sin t - ab\sin^2 t - ab\cos^2 t + b^2 \sin t\cos t \right] dt =$$

$$= -ab\int_0^{2\pi} 1\, dt + \int_0^{2\pi} (b^2 - a^2)\frac{1}{2}\sin 2t\, dt = -2\pi ab + 0 = \underline{\underline{-2\pi ab}}$$

$$\underline{6.7}\quad A = \int_{-\pi}^{\pi} c_x(t)\, c_y'(t)\, dt = \int_{-\pi}^{\pi} \underset{v'}{\sin t} \cdot \underset{u}{t}\, dt =$$

$$= \underset{u\ \ v}{\Big[t(-\cos t) \Big]_{-\pi}^{\pi}} - \underbrace{\int_{-\pi}^{\pi} \underset{u'}{1} \cdot \underset{v}{(-\cos t)}\, dt}_{0} = \Big[\pi - (-\pi) \Big] = 2\pi$$