8.1) $f(\vec{x}) = A\vec{x} + \vec{b}$ $\quad \bar{f}(\vec{v}) = A\vec{v}$

$$A\begin{pmatrix}1 & 1\\ 2 & 0\end{pmatrix} = \begin{pmatrix}-7 & 1\\ -2 & -2\end{pmatrix} \Rightarrow A\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & -4\\ -2 & 0\end{pmatrix} = A$$

$$\begin{bmatrix}-3\\ 3\end{bmatrix} = \begin{pmatrix}1 & -4\\ -2 & 0\end{pmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} + \vec{b} \Rightarrow \begin{pmatrix}1\\ 3\end{pmatrix} = \vec{b}$$

8.2) $f_1: \begin{bmatrix}0\\ 0\end{bmatrix} \mapsto \begin{bmatrix}6\\ 7\end{bmatrix}$, $\begin{bmatrix}1\\ 0\end{bmatrix} \mapsto \begin{bmatrix}4\\ 5\end{bmatrix}$, $\begin{bmatrix}0\\ 1\end{bmatrix} \mapsto \begin{bmatrix}7\\ 12\end{bmatrix}$

$f_2: \begin{bmatrix}0\\ 0\end{bmatrix} \mapsto \begin{bmatrix}-1\\ +1\end{bmatrix}$, $\begin{bmatrix}1\\ 0\end{bmatrix} \mapsto \begin{bmatrix}1\\ -1\end{bmatrix}$, $\begin{bmatrix}0\\ 1\end{bmatrix} \mapsto \begin{bmatrix}2\\ 0\end{bmatrix}$

$$f = f_1 \circ f_2^{-1}$$

$$f^{-1} = f_2 \circ f_1^{-1}$$

$$\Rightarrow f_1(\vec{x}) = \begin{pmatrix}-2 & +1\\ -2 & +5\end{pmatrix}\vec{x} + \begin{pmatrix}6\\ 7\end{pmatrix}$$

$$f_2(\vec{x}) = \begin{pmatrix}2 & 3\\ -2 & -1\end{pmatrix}\vec{x} + \begin{pmatrix}-1\\ +1\end{pmatrix}$$

$$f_1^{-1}(\vec{x}) = \frac{-1}{8}\begin{pmatrix}+5 & -1\\ +2 & -2\end{pmatrix}\left(\vec{x} - \begin{pmatrix}6\\ 7\end{pmatrix}\right)$$

$$f_2^{-1}(\vec{x}) = \frac{1}{4}\begin{pmatrix}-1 & -3\\ 2 & 2\end{pmatrix}\left(\vec{x} - \begin{pmatrix}-1\\ 1\end{pmatrix}\right)$$

$$f_1^{-1}(\vec{x}) = \frac{-1}{8}\begin{pmatrix}+5 & -1\\ +2 & -2\end{pmatrix}\left(\vec{x} - \begin{pmatrix}6\\7\end{pmatrix}\right) \qquad f_1(\vec{x}) = \begin{pmatrix}-2 & +1\\ -2 & +5\end{pmatrix}\vec{x} + \begin{pmatrix}6\\7\end{pmatrix}$$

$$f_2^{-1}(\vec{x}) = \frac{1}{4}\begin{pmatrix}-1 & -3\\ 2 & 2\end{pmatrix}\left(\vec{x} - \begin{pmatrix}-1\\1\end{pmatrix}\right) \qquad f_2(\vec{x}) = \begin{pmatrix}2 & 3\\ -2 & -1\end{pmatrix}\vec{x} + \begin{pmatrix}-1\\+1\end{pmatrix}$$

$$f^{-1}(\vec{x}) = f_2 \circ f_1^{-1}(\vec{x}) = \begin{pmatrix}2 & 3\\ -2 & -1\end{pmatrix}\left(\frac{-1}{8}\begin{pmatrix}+5 & -1\\ +2 & -2\end{pmatrix}\left(\vec{x} - \begin{pmatrix}6\\7\end{pmatrix}\right)\right) + \begin{pmatrix}-1\\+1\end{pmatrix} =$$

$$= \frac{-1}{8}\begin{pmatrix}+16 & -8\\ -12 & 4\end{pmatrix}\left(\vec{x} - \begin{pmatrix}6\\7\end{pmatrix}\right) + \begin{pmatrix}-1\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}-4 & 2\\ 3 & -1\end{pmatrix}\vec{x} - \frac{1}{2}\begin{pmatrix}10\\11\end{pmatrix} + \begin{pmatrix}-1\\1\end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix}-4 & 2\\ 3 & -1\end{pmatrix}\vec{x} + \begin{pmatrix}4\\-\frac{9}{2}\end{pmatrix}$$

$$f(\vec{x}) = f_1 \circ f_2^{-1}(\vec{x}) = \begin{pmatrix}-2 & 1\\ -2 & 5\end{pmatrix}\left(\frac{1}{4}\begin{pmatrix}-1 & -3\\ 2 & 2\end{pmatrix}\left(\vec{x} - \begin{pmatrix}-1\\1\end{pmatrix}\right)\right) + \begin{pmatrix}6\\7\end{pmatrix} =$$

$$= \frac{1}{4}\begin{pmatrix}+4 & 8\\ 12 & 16\end{pmatrix}\left(\vec{x} - \begin{pmatrix}-1\\1\end{pmatrix}\right) + \begin{pmatrix}6\\7\end{pmatrix} = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\vec{x} + \begin{pmatrix}-1\\-1\end{pmatrix} + \begin{pmatrix}6\\7\end{pmatrix} = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\vec{x} + \begin{pmatrix}5\\6\end{pmatrix}$$

8.3) $f(\vec{x}) = A\vec{x} + \vec{b} \Rightarrow \bar{f}(\vec{x} - \vec{y}) = A(\vec{x} - \vec{y})$

$$\bar{f}\left(\begin{pmatrix}2\\1\\-1\end{pmatrix} - \begin{pmatrix}0\\1\\1\end{pmatrix}\right) = \begin{pmatrix}1\\3\end{pmatrix} - \begin{pmatrix}+1\\-1\end{pmatrix} \qquad \bar{f}\begin{pmatrix}2\\0\\-2\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix}$$

$$\bar{f}\left(\begin{pmatrix}1\\0\\2\end{pmatrix} - \begin{pmatrix}0\\1\\1\end{pmatrix}\right) = \begin{pmatrix}0\\2\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix} \qquad \bar{f}\begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$$

$$\bar{f}\left(\begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}0\\1\\1\end{pmatrix}\right) = \begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix} \qquad \bar{f}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\2\end{pmatrix}$$

$$A = \begin{pmatrix}0 & 1 & 0\\2 & -1 & 0\end{pmatrix}$$

$$\bar{f}\begin{pmatrix}3\\1\\-2\end{pmatrix} = \begin{pmatrix}1\\5\end{pmatrix}$$

$$\vec{b} = A\begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}+1\\+1\end{pmatrix} - \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$f\begin{pmatrix}3\\1\\-2\end{pmatrix} = A\begin{pmatrix}3\\1\\-2\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}1\\5\end{pmatrix} \equiv \begin{bmatrix}1\\5\end{bmatrix}$$

8.4) $\bar{f}$ vzhledem ke kan. bázi:

$$\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}^{-1} = \qquad // \; -\frac{1}{2}\begin{pmatrix} 6 & -5 \\ -4 & 3 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}}_{\begin{pmatrix} 19 & 9 \\ 24 & 8 \end{pmatrix}}$$

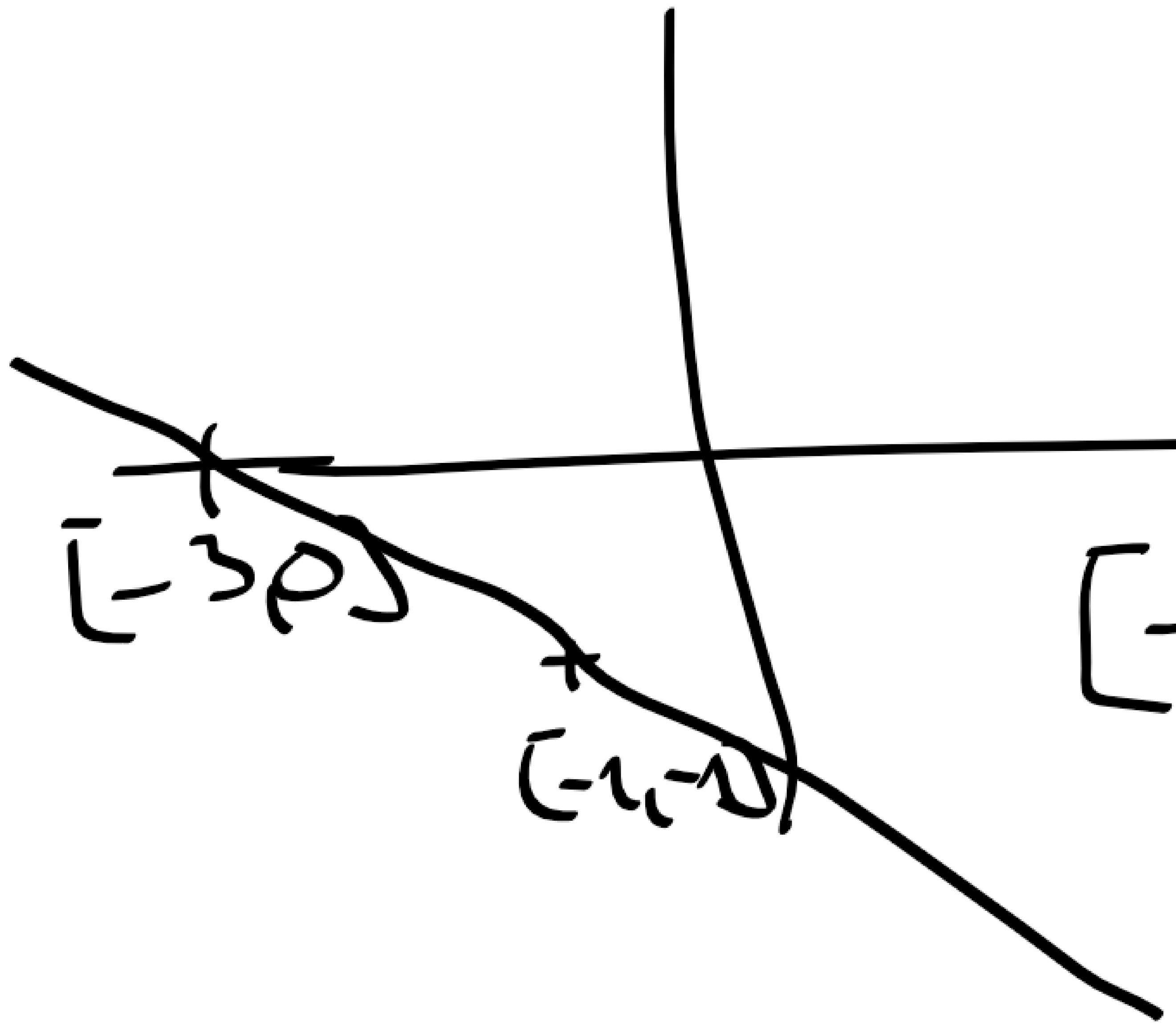
$$f\left(\begin{bmatrix} 3 \\ 4 \end{bmatrix}\right) = A'\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \vec{b} = A'\left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right] + A'\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \vec{b} =$$

$$= \bar{f}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + f\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{pmatrix} 19 & 7 \\ 24 & 8 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} + \begin{bmatrix} 5 \\ -8 \end{bmatrix} =$$

$$= \begin{pmatrix} -12 \\ -16 \end{pmatrix} + \begin{bmatrix} 5 \\ -8 \end{bmatrix} = \begin{bmatrix} -7 \\ -24 \end{bmatrix}$$

8.5) $x+2y+3=0$

$\begin{bmatrix}-3\\0\end{bmatrix}, \begin{bmatrix}-1\\-1\end{bmatrix}$



$[-1,-1]+[1,2]=[0,1]$

$\pi(\vec{x}) = A\vec{x} + \vec{b}$

$\pi\left(\begin{bmatrix}-1\\-1\end{bmatrix}\right) = \begin{bmatrix}-1\\-1\end{bmatrix}$

$\pi\left(\begin{bmatrix}-3\\0\end{bmatrix}\right) = \begin{bmatrix}-3\\0\end{bmatrix}$

$\pi\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\-1\end{bmatrix}$

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$A\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$A\begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix}$

$A = \begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}^{-1}$

$= \begin{pmatrix}0 & 2\\0 & -1\end{pmatrix}\begin{pmatrix}-1 & -2\\-2 & 1\end{pmatrix}\left(-\frac{1}{5}\right) = \frac{1}{5}\begin{pmatrix}4 & -2\\-2 & 1\end{pmatrix}$

$\frac{1}{5}\begin{pmatrix}4 & -2\\-2 & 1\end{pmatrix}\begin{pmatrix}-1\\-1\end{pmatrix} + \vec{b} = \begin{pmatrix}-1\\-1\end{pmatrix}$

$\vec{b} = \begin{pmatrix}-1\\-1\end{pmatrix} - \frac{1}{5}\begin{pmatrix}-2\\1\end{pmatrix} = \frac{1}{5}\begin{pmatrix}-3\\-6\end{pmatrix}$

$\pi^2(\vec{x}) = A(A\vec{x}+\vec{b}) + \vec{b} =$

$= A^2\vec{x} + (A+I_2)\vec{b} = A\vec{x}$

$+ \frac{1}{5}\begin{pmatrix}9 & -2\\-2 & 6\end{pmatrix}\frac{1}{5}\begin{pmatrix}-3\\-6\end{pmatrix} = A\vec{x} + \frac{1}{5^2}\begin{pmatrix}-15\\-30\end{pmatrix}$

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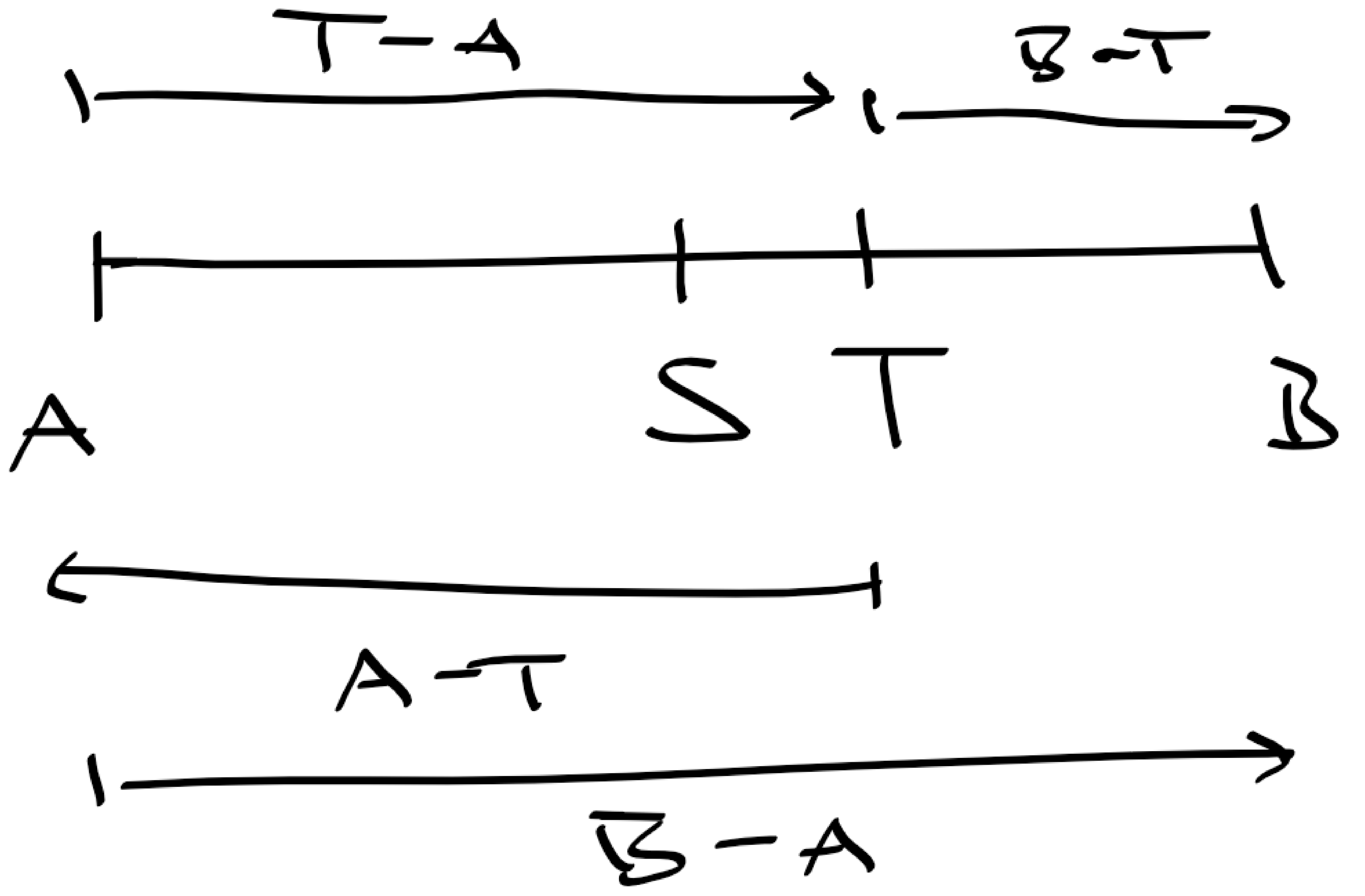
8.6) PARAM: $\begin{bmatrix}2\\2\end{bmatrix} + t\begin{bmatrix}2-1\\3-4\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix} + t\begin{bmatrix}1\\4\end{bmatrix}$

ROVNICE: $x + y + 1 = 0$

8.7)



$$\frac{AT}{TB} = 2 \qquad 2(B-T) = (T-A)$$

$$\frac{TA}{AB} = \frac{2}{3} \qquad \frac{2}{3}(B-A) = (A-T)$$

$$\frac{ST}{TB} = \frac{1}{2} \qquad \frac{1}{2}(B-T) = (T-S)$$

$$S = \frac{1}{2}A + \frac{1}{2}B$$

$$T = \frac{1}{3}A + \frac{2}{3}B \Rightarrow B = \frac{3}{2}\left(T - \frac{1}{3}A\right) = \frac{3}{2}T - \frac{1}{2}A$$

$$S = \frac{1}{2}A + \frac{1}{2}\left(\frac{3}{2}T - \frac{1}{2}A\right) = \underline{\underline{\frac{1}{4}A + \frac{3}{4}T}}$$