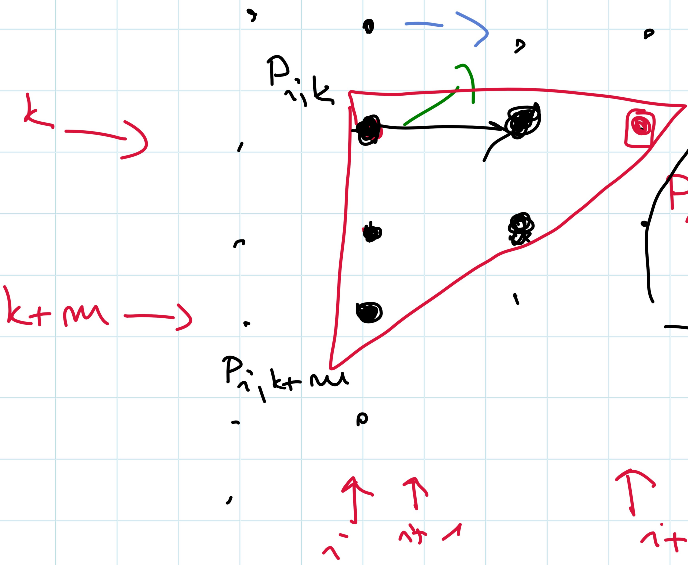
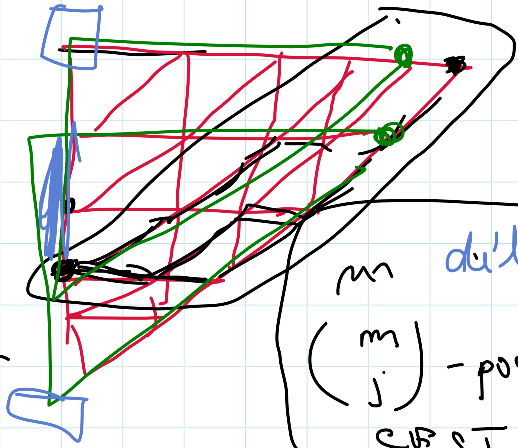
LEMMA 2.10

$\boxed{t \ldots \text{pevné}}$



$$\boxed{P_{i+m,k} = \sum_{j=0}^{m} P_{i,k+j} \cdot B_j^m(t)}$$



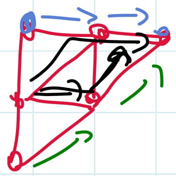
m dílka

$\binom{m}{j}$ - počet CEST

PŘ $\quad m = 2$

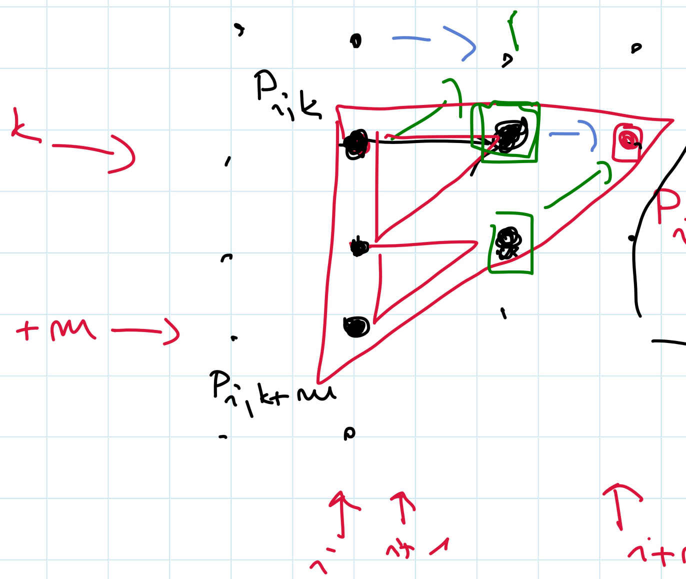
$$\left(P_{i,k}(1-t) + P_{i,k+1} \cdot t\right)(1-t) + \left(P_{i,k+1}(1-t) + P_{i,k+2} \cdot t\right) \cdot t =$$

$$= P_{i,k}(1-t)^2 + 2P_{i,k+1}\, t(1-t) + P_{i,k+2} \cdot t^2$$



LEMMA 2.10

(t ... pevné)



$$\boxed{P_{i+m,k} = \sum_{j=0}^{m} P_{i,k+j} \cdot B_j^m(t)}$$

Dk: indukce podle m

1) $m = 0$

$$P_{i,k} = \sum_{j=0}^{0} P_{i,k+j} B_j^0(t) = P_{i,k} B_0^0(t) = P_{i,k}$$

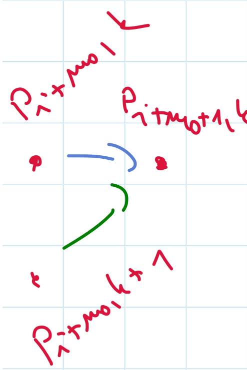
$$\boxed{B_0^0(t) \equiv 1}$$

$m = 1$

$$P_{i+1,k} = \sum_{j=0}^{1} P_{i,k+j} B_j^1(t) =$$

$$= P_{i,k} \cdot \underbrace{B_0^1(t)}_{(1-t)} + P_{i,k+1} \cdot \underbrace{B_1^1(t)}_{t} =$$

Platí podle konstrukce



2) Předpokládejme platnost pro $m = m_0$ a ukaž pro $m = m_0 + 1$

konstrukce

$$P_{i+m_0+1,k} = P_{i+m_0,k}(1-t) + P_{i+m_0,k+1}\, t =$$

ind. př.

$$= \Big[\sum_{j=0}^{m_0} P_{i,k+j} \cdot B_j^{m_0}(t)\Big](1-t) + \Big[\sum_{j=0}^{m_0} P_{i+m_0,k+1+j} B_j^{m_0}(t)\Big] t$$

$w = j$ $\qquad$ $w = 1+j$, $j = w-1$, $j = 0_{m_0+1}$

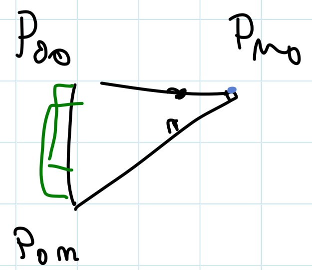
$$= \sum_{w=0}^{m_0+1} P_{i,k+w}\Big[B_w^{m_0}(t)(1-t) + B_{w-j}^{m_0}(t) \cdot t\Big] = \sum_{w=0} P_{i,k+w} B_w^{m_0+1}(t)$$

DŮKAZ ALGORITMU

BOD v lemmatu

$$\boxed{\begin{matrix} m = M \\ k \to 0 \\ i \to 0 \end{matrix}}$$

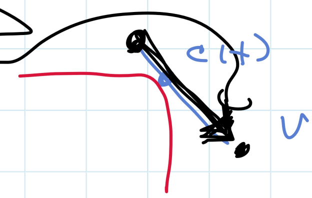
(s stupni)



$$P_{M,0} = \sum_{j=0}^{M} P_{0,j} \cdot B_j^M(t) = c(t)$$

TEČNA

BODY $P_{M-1,0}$, $P_{M-1,1}$ a $c(t)$ leží na úsečce



DOTYK??

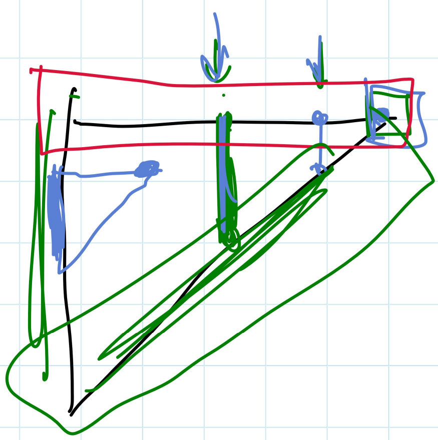
podle LEMMA

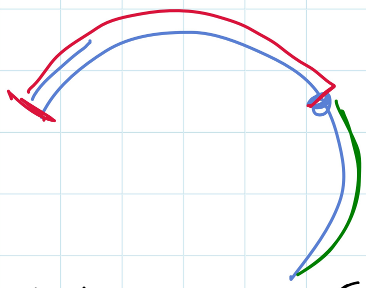
$c'(t)$ má stejný směr $(P_{M-1,1} - P_{M-1,0})$

$$P_{M-1,1} - P_{M-1,0} = \sum_{j=0}^{M-1} P_{0,j+1} B_j^{M-1}(t) - \sum_{j=0}^{M-1} P_{0,j} B_j^{M-1}(t)$$

$\nu = j$ $\quad$ $\nu = j$

$$= \sum_{\nu=0}^{M-1} \underbrace{(P_{0,\nu+1} - P_{0,\nu})}_{\frac{1}{M} Q_j} B_j^{M-1}(t) = \frac{1}{M} \cdot c'(t)$$



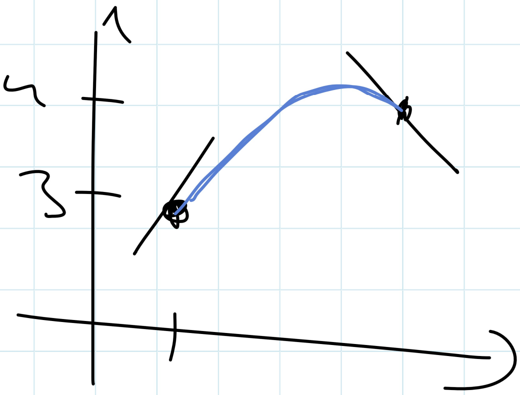


t

$u = s \cdot t$

$c(u)$ $\quad \ldots \quad u \in (0,t)$

$c_1(s)$ $\quad s \in (0,1)$



$f(1) = 3$

$f'(2) = 1$

$f'(3) = -1$

$f(4) = 5$