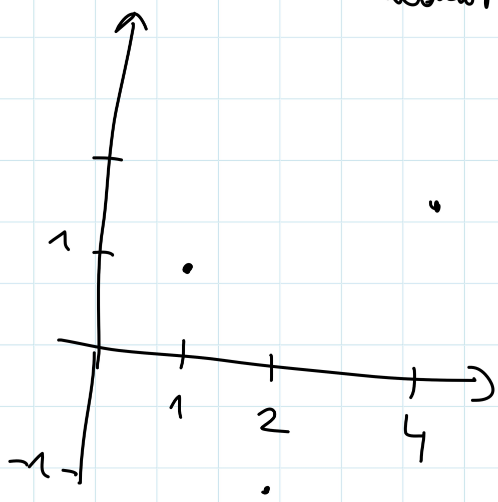
LAGRANGE

$n+1$ bodů x_i

$n+1$ hodnot f_i

$\text{st}\, f \leq n$



$f(1) = 1$

$f(2) = -1$

$f(4) = 2$

$$\boxed{f(x) = a_0 + a_1 x + a_2 x^2}$$

$f(x_i) = f_i$

$f(2) = -1$

$$a_0 + a_1(2) + a_2(2)^2 = -1$$

$$\begin{array}{ccc} \begin{pmatrix} 1 & 2 & 2^2 \end{pmatrix} & \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} & = -1 \quad f_i \\ x_i^0 \quad x_i^1 \quad x_i^2 & & \end{array}$$

$$\boxed{\underbrace{\begin{pmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & & & x_1^n \\ & & & & \\ 1 & x_n & \dots & \dots & x_n^n \end{pmatrix}}_{M} \begin{pmatrix} a_0 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} f_0 \\ \\ f_n \end{pmatrix}}$$

$$\det M = \prod_{i \neq j} (x_i - x_j) \neq 0 \Rightarrow \text{regulární}$$

$\Rightarrow \exists!$ řešení

$$l_i(x_j) = \delta_{ij} \begin{cases} 1 \\ 0 \end{cases}$$

$$f(x) = \sum f_i \cdot l_i(x) \qquad f(x_j) = \sum f_i \delta_{ij} = f_j$$

"Ik"

$l_i(x_j)$ $i \neq j$

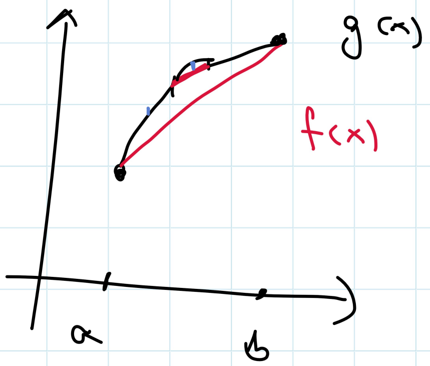
$\Rightarrow (x - x_j)$ v čitateli l_i

$\Rightarrow l_i(x_j) = 0$

$l_i(x_i)$

$\Rightarrow$ čitatel a jmenovatel l_i jsou stejné

$g''(x)$ ji ppojito' na $[a,b]$ $\Rightarrow K = \max_{x\in[a,b]} |g''(x)|$

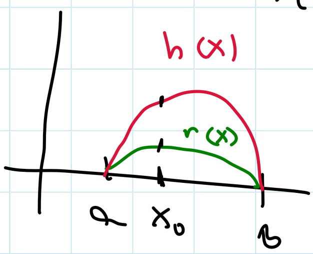


$$\max_{x\in[a,b]} |g(x)-f(x)| \le (b-a)^2 \cdot \frac{K}{8}$$

Dk.

$r(x) = g(x) - f(x)$ $\qquad r(a) = r(b) = 0$

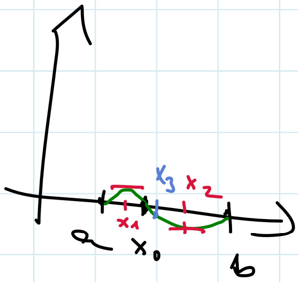
$h(x) = (x-a)(b-x)$ $\qquad h(a) = h(b) = 0$



$= -x^2 + x(b+a) - ab$ $\qquad \max_{x\in[a,b]} h(x) = h\left(\frac{a+b}{2}\right) = \frac{(b-a)^2}{4}$

$x_0 \in (a,b)$ $\qquad F(x) = r(x) - L\,h(x)$ $\qquad L = \frac{r(x_0)}{h(x_0)}$

$F(x_0) = 0 = F(a) = F(b)$



(1) Rollovo vate $\quad \exists\, x_1 \in (a, x_0)$; $x_2 \in (x_0, b)$

$F'(x_1) = F'(x_2) = 0$ $\qquad C^1$

(2) Rolleovo v. na funkcio $F'(x)$

$\exists\, x_3 \in (x_1, x_2) : F''(x_3) = 0.$

$$0 = F''(x_3) = r''(x_3) - L\,h''(x_3) = g''(x_3) - \underbrace{f''(x_3)}_{0} - L\,\underbrace{h''(x_3)}_{-2} =$$

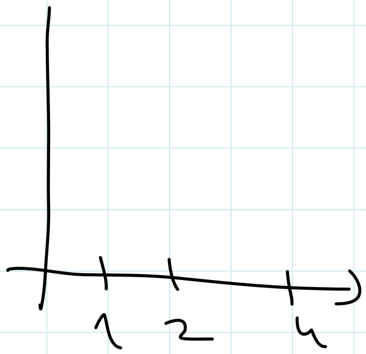
$$= g''(x_3) - 0 + L\cdot 2$$

$$L = \frac{g''(x_3)}{2} \Rightarrow |L| \le \frac{K}{2}$$

$$L = \frac{r(x_0)}{h(x_0)} \Rightarrow \quad r(x_0) = L\cdot h(x_0) \le h(x_0)\cdot\frac{K}{2} \le \frac{(b-a)^2}{4}\cdot\frac{K}{2}$$

$\blacksquare$

$\underline{\underline{n=2}}$

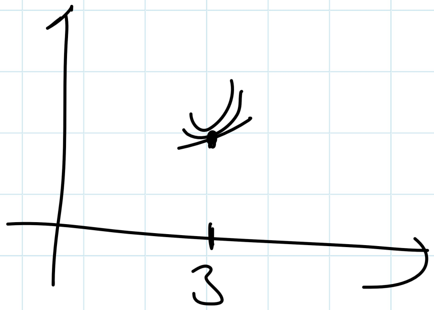


$$f(1)=1$$
$$f'(2)=11$$
$$f''(4)=4$$

$$f(x)= a_0 + a_1 x + a_2 x^2$$
$$f'(x) = a_1 + 2a_2 x \qquad f''(x) = 2a_2$$

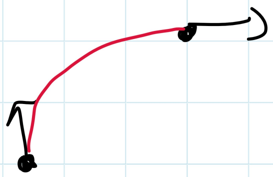
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 11 \\ 4 \end{pmatrix}$$

$l_0(x)$ $\qquad l_0(1)=\underline{1}$ $\qquad l_0'(2)=\underline{0}$ $\qquad l_0''(4)=\underline{0}$



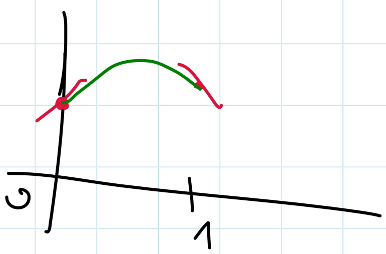
$$f(3)$$
$$f'(3)$$
$$f''(3)$$
$$f'''(3)$$

$\underline{\underline{C^1}}$ Hermitova interpolácia na $[0,1]$



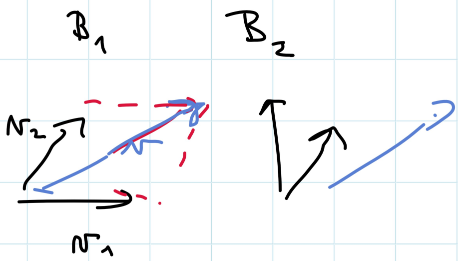
x ... zľava

y ... zľava



$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$
$$f'(x) = a_1 + 2a_2 x + 3a_3 x^2$$

$$f'(0) = a_1$$
$$f'(1) = \underline{a_1 + 2a_2 + 3a_3} \,//$$



$$[v]_{B_1} = \begin{pmatrix} 1,2 \\ 1,3 \end{pmatrix}$$

$$[v]_{B_2}$$

$$[id]^{B_1}_{B_2} \cdot [v]_{B_1} = [v]_{B_2}$$

$$[v_1]_B$$

$$\parallel$$

$$[id]^{B_1}_{B_2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = [v_1]_{B_2}$$

$$\left([v_1]_{B_2} \,\middle|\, [v_2]_{B_2} \right)$$

$$\boxed{f = a_0 + a_1 x + a_2 x^2 + a_3 x^3}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{pmatrix}}_{[id]^M_R} \cdot \underbrace{\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}}_{= [f]_M} = \underbrace{\begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix}}_{= [f]_R}$$

$$(1, x, x^2, x^3) \cdot$$

$$[id]^R_M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{pmatrix}$$

$$[h_0(x)]_M$$

$$h_0(x) = 1 - 3x^2 + 2x^3$$