$$\underbrace{tg\,(xy^2+x) - \log(x+y)}_{F(x,y)} = 0 \qquad [x_0, y_0] = [0,1]$$

- $F(0,1) = tg\,0 - \log 1 = 0$
- $F \in C^\infty(U)$ pro $U \ni [0,1]$ vhodné

$$\frac{dF}{dx}\Big|_{(0,1)} = \left(\frac{1}{\cos^2(xy^2+x)} \cdot (y^2+1) - \frac{1}{x+y} \cdot 1\right)\Big|_{(0,1)} =$$

$$= \frac{1}{1} \cdot (1+1) - \frac{1}{1} \cdot 1 = 1 \neq 0$$

VOIF $\Rightarrow$ $\exists U \ni 1 \; \exists V \ni 0 \; \forall y \in U \; \exists!\, x \in V: F(x,y) = 0$

a funkce $y \mapsto x(y)$ je třídy C^∞ na U

- $0 = F(x(y), y)$ na U

$$\Rightarrow 0 = \frac{1}{\cos^2(xy^2+x)}\left(x'y^2 + x\,2y + x'\right) - \frac{1}{x+y}(x'+1)$$

$\overset{(0,1)}{\Rightarrow}$ $0 = 1 \cdot (x'(1) \cdot 1 + 0 + x'(1)) - \frac{1}{1}(x'(1)+1)$

$0 = x'(1) - 1$

$x'(1) = 1$

$\Rightarrow$ $x' > 0$ na nějakém okolí 1 $\Rightarrow$ x rostoucí na tomto okolí

$\Rightarrow$ x zde nemá extrém

- předpoklady 2 + 2
- použití VOIF 2
- derivace 5
- dosazení 2
- úvaha 3

$f = x^2 + z^2 + 2xz = (x+z)^2$ spojitá (+1)

$M = \{ \cancel{x,y} \ 0 \le z \le 1 - x^2 - y^2 \}$ uzavřená

$\Rightarrow x^2 + y^2 \le 1 \Rightarrow M$ kompaktní (+2)

$0 \le z \le 1$

- $M_1 = \{ 0 < z < 1 - x^2 - y^2 \}$, $Df = (2(x+z), 0, 2(x+z)) = 0 \iff z = -x, y \in \mathbb{R}$

(+2) $\Rightarrow$ pro $P = \{ [t, s, -t] : t, s \in \mathbb{R} \}$ máme $P \cap M_1$ jsou body podezřelé z extrému

- $M_2 = \{ 0 = z, z < 1 - x^2 - y^2 \} = \{ x^2 + y^2 < 1 \}$, $g(x,y) = x^2$ (+2)

$\Rightarrow$ $[0, y, 0]$, $y \in (-1, 1)$

- $M_3 = \{ z > 0, z = 1 - x^2 - y^2 \}$, $g(x,y) = (x + 1 - x^2 - y^2)^2$ (+3)

$\Rightarrow \left(Dg(x,y) = 2(x + 1 - x^2 - y^2)(-2x + 1, -2y) \right) = 0$

$\iff x = -z$ nebo $(x,y) = [1/2, 0]$

$(x, y, z) = [1/2, 0, \frac{3}{4}]$

- $M_4 = \{ z = 0, z = 1 - x^2 - y^2 \} = \{ x^2 + y^2 = 1 \}$, $g(x,y) = x^2$ (+3)

$\Rightarrow [\pm 1, 0, 0]$, $[0, \pm 1, 0]$

Závěr: • maximum v $[1/2, 0, 3/4]$ (+2)

• minimum v $P \cap M = \{ 0 \le -x, 0 \le 1 + x - x^2 - y^2 \} =$

$= \{ x \le 0, x^2 + y^2 - x \le 1 \} = \{ x \le 0, (x - 1/2)^2 + y^2 \le \frac{5}{4} \}$

$\Rightarrow P \cap M = \{ [x, y, -x] : x \le 0, (x - 1/2)^2 + y^2 \le 3/4 \}$

$$\begin{pmatrix} 3 & 1 & 1 & 4 \\ a & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & -20 & -50 & -5 \\ 0 & 4-7a & 10-17a & 1-3a \\ 0 & -12 & -30 & -3 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \\ 0 & 4-7a & 10-17a & 1-3a \\ 0 & 4 & 10 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \\ 0 & 4-7a & 10-17a & 1-3a \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ \\ - \\ \\ \end{matrix} \sim$$

$$\sim \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \\ 0 & -7a & -17a & -3a \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{matrix} a = 0 \Rightarrow h(A) = 2 \\ a \neq 0 \Rightarrow h(A) \ldots \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \\ 0 & -7 & -17 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ \cdot 7 \\ \cdot 4 \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \\ 0 & 0 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow h(A) = 3$$

- det $A = 0$, neboť A ~~je singu~~ hodnost $A = 2$

- úpravy 9
- $a = 0$ 2
- $a \neq 0$ 2
- determinant 2

$$\sum (-1)^n \underbrace{(\operatorname{arccotg} n) \frac{n}{n^p+1}}_{a_n}$$

- $p=0$: $a_n = \dfrac{\operatorname{arccotg} n}{\frac{1}{n}} \cdot \dfrac{1}{n} \cdot \dfrac{n}{1+1} = \longrightarrow 1/2 \Rightarrow \sum (-1)^n a_n$ diverguje

- $p>0$: $a_n = \dfrac{\operatorname{arccotg} n}{1/n} \cdot \dfrac{1}{n^p+1} \Rightarrow$ pro $b_n = \dfrac{1}{n^p}$ máme

A.K.

$$\frac{a_n}{b_n} \longrightarrow 1 \in (0,\infty) \Rightarrow \left(\sum a_n \text{ konv.} \Leftrightarrow \sum \frac{1}{n^p} \text{ konv.} \Leftrightarrow p>1\right)$$

- $p \in (0,1]$: $f(x) = (\operatorname{arccotg} x) \dfrac{x}{x^p+1} \Rightarrow$

$$f'(x) = \frac{-1}{1+x^2} \cdot \frac{x}{x^p+1} + \operatorname{arccotg} x \, \frac{x^p+1 - x p x^{p-1}}{(x^p+1)^2} =$$

$$= \frac{-x}{(x^2+1)(x^p+1)} + \operatorname{arccotg} x \, \frac{x^p(1-p)+1}{(x^p+1)^2}$$

$$= \frac{1}{x^p+1} \left[\frac{-x}{x^2+1} + \frac{\operatorname{arccotg} x \, (1+(1-p)x^p)}{(x^p+1)} \right]$$

$$= \frac{1}{(x^p+1)^2 (x^2+1)} \left[-x(x^p+1) + (x^2+1) \operatorname{arccotg} x \, (1+(1-p)x^p) \right]$$

$$= -//- \left[x^{p+1} \underbrace{\left(-1 - x^{-p} + \frac{\operatorname{arccotg} x}{1/x} \cdot \frac{x^2+1}{x^2} \left(x^{-p} + (1-p)\right) \right)}_{} \right]$$

$$\xrightarrow[x\to\infty]{} -1 - 0 + 1 \cdot 1 \cdot (0 + (1-p)) =$$

$$= -1 + 1 - p = -p < 0$$

$\Rightarrow f' < 0$ od jistého $x_0 \Rightarrow a_n \searrow$, zjevně $a_n \to 0$

$\Rightarrow \sum (-1)^n a_n$ konverguje dle Leibnize

- $p=0$ 2
- $p>0$
  - nalezení řady 2
  - limita 2
  - závěr 2
- $p \in (0,1\rangle$
  - f' 3
  - odhad f' 3
  - závěr 1