$\cos xy = \sqrt{xy+1}$, $[x_0, y_0] = [1,0]$

- $F(x,y) = \cos xy - \sqrt{xy+1}$ je třídy C^∞ na nějakém okolí $[1,0]$ +1
- $F(1,0) = \cos 0 - \sqrt{0+1} = 0$ +1
- $\frac{\partial F}{\partial y}(1,0) = \left(-(\sin xy)\,x - \frac{x}{2\sqrt{xy+1}}\right)\Big|_{(x,y)=(1,0)} = -\frac{1}{2\sqrt{1}} = -\frac{1}{2} \neq 0$ +3

VoIF
$\Rightarrow \exists\, U \ni 1\ \exists\, V \ni 0\ \forall x \in U\ \exists!\, y \in V:\ F(x,y) = 0$. Tato funkce je třídy C^∞. +3

Tedy $\cos x\varphi(x) - \sqrt{x\varphi(x)+1} = 0$, $x \in U$

$$-(\sin x\varphi(x))(\varphi(x) + x\varphi'(x)) - \frac{\varphi(x) + x\varphi'(x)}{2\sqrt{x\varphi(x)+1}} = 0 \quad +4$$

$x=1$:
$\varphi(1)=0$

$$-\sin 0\,(0 + 1\cdot\varphi'(1)) - \frac{0 + 1\varphi'(1)}{2\sqrt{0+1}} = 0$$

$$\frac{\varphi'(1)}{2} = 0$$

$$\varphi'(1) = 0 \quad +3$$

$f = (x^2+2y^2)e^{-2x^2-y^2}$, $M = \{x \geq 0, y \geq 0\}$.

$\nabla f = \big(2x\, e^{-2x^2-y^2} + (x^2+2y^2)e^{-2x^2-y^2}(-4x),$

$4y\, e^{-2x^2-y^2} + (x^2+2y^2)e^{-2x^2-y^2}(-2y)\big)$

+2

$= e^{-2x^2-y^2}\big(2x - 4x(x^2+2y^2),\ 4y - 2y(x^2+2y^2)\big)$

· $\nabla f = 0$ pro $(x,y) \in \operatorname{Int} M \iff \begin{array}{l} 1-2(x^2+2y^2)=0 \\ 2-1(x^2+2y^2)=0 \end{array}$ +2 nemá řešení

· $M_1 = \{[x,0] : x \geq 0\}$, $g(x) = x^2 e^{-2x^2}$, pak $g(0)=0$, $g(x) \xrightarrow[x\to\infty]{} 0$

$g'(x) = 2x e^{-2x^2} + x^2 e^{-2x^2}(-4x) = e^{-2x^2}(2x)(1-2x^2) = 0$

$\iff x = \frac{1}{\sqrt{2}}$ +3

· $M_2 = \{[0,y] : y \geq 0\}$, $g(y) = 2y^2 e^{-y^2}$, pak $g(0)=0$, $g(y) \xrightarrow[y\to\infty]{} 0$

$g'(y) = 2(2y e^{-y^2} + y^2 e^{-y^2}(-2y)) = 4y e^{-y^2}(1-y^2) = 0$

$\iff y = 1$ +3

$\Rightarrow \underbrace{[\tfrac{1}{\sqrt{2}}, 0]}_{\alpha}, \underbrace{[0,1]}_{\beta}$, pak $f(\alpha) = \frac{1}{2}e^{-2\cdot\frac{1}{2}} = \frac{1}{2}e^{-1}$

$f(\beta) = 2\cdot 1\, e^{-1} = 2e^{-1}$

· $f(x,y) \leq 2(x^2+y^2)e^{-x^2-y^2} = 2(x^2+y^2)e^{-(x^2+y^2)}$

Jelikož $re^{-r} \xrightarrow[r\to\infty]{} 0$, existuje $r_0 > 0$, že pro $r \geq r_0$ platí $2re^{-r} < f(\beta)$. Pak

Pak $N = M \cap B(0, r_0)$ je kpt. & $\beta \in N$.

+5

- $f < f(\beta)$ na $M \setminus N$
- $\max f(N) = f(\beta)$: · uvnitř N není podezřelý bod
  · na (kružnici o poloměru $r_0 \cap M$) též platí $f < f(\beta)$

$\Rightarrow \max f(M) = f(\beta)$, jelikož $f(0,0) = 0$ je $\min f(M) = 0$.

$$
\left(\begin{array}{cccc|c}
7 & 1 & 3 & -11 & k \\
3 & 0 & 0 & -2 & 2 \\
0 & 0 & -3 & 7 & 8 \\
1 & -1 & 0 & 0 & 0
\end{array}\right)
\overset{+1}{\sim}
\left(\begin{array}{cccc|c}
1 & -1 & 0 & 0 & 0 \\
3 & 0 & 0 & -2 & 2 \\
0 & 0 & -3 & 7 & 8 \\
7 & 1 & 3 & -11 & \varepsilon
\end{array}\right)
$$

$$
\overset{+4}{\sim}
\left(\begin{array}{cccc|c}
1 & -1 & 0 & 0 & 0 \\
0 & 3 & 0 & -2 & -2 \\
0 & 0 & -3 & 7 & 8 \\
0 & 8 & 3 & -11 & k
\end{array}\right)
\overset{+3}{\sim}
\left(\begin{array}{cccc|c}
1 & -1 & 0 & 0 & 0 \\
0 & 3 & 0 & -2 & 2 \\
0 & 0 & -3 & 7 & 8 \\
0 & 0 & 3 & \frac{-17}{3} & \frac{3\varepsilon-16}{3}
\end{array}\right)
$$

$$
\overset{+3}{\sim}
\left(\begin{array}{cccc|c}
1 & -1 & 0 & 0 & 0 \\
0 & 3 & 0 & -2 & 2 \\
0 & 0 & -3 & 7 & 8 \\
0 & 0 & 0 & 4/3 & \varepsilon + 8/3
\end{array}\right)
$$

$x_4 = \frac{3}{4} k + 2$

$x_3 = \frac{7}{4} \varepsilon + 2$ $\qquad\qquad +4$

$x_2 = \frac{1}{2} \varepsilon + 2$

$x_1 = \frac{1}{2} k + 2 \quad , \varepsilon \in \mathbb{R}$

$\sum (-1)^n \underbrace{\left(\frac{e^{1/n^p}-1}{n^{5p}}\right)}_{a_n}$, pak $a_n \geq 0$. $p=0$ ... ~~konverguje absolutně~~ diverguje +1

$p>0$:

· A.K.: $\sum a_n$ srovnáme s $b_n = \sum \frac{1}{n^{6p}}$, pak +3

$$\frac{a_n}{b_n} = \frac{e^{1/n^p}-1}{n^{5p}} \cdot n^{6p} = \frac{e^{1/n^p}-1}{1/n^p} \xrightarrow[n\to\infty]{} 1$$ +3

$$\Rightarrow \left(\sum a_n \text{ konv.} \Leftrightarrow \sum \frac{1}{n^{6p}} \text{ konv.} \Leftrightarrow \begin{matrix} 6p>1 \\ p>1/6 \end{matrix}\right)$$ +1

· N.A.K.: $\frac{1}{n^p} = x \Rightarrow$ máme funkci $x^5(e^x-1)$, kt. je pro $x>0$ rostoucí, +4

tedy $a_n > a_{n+1}$, $n \in \mathbb{N}$. Dále

$$\frac{e^{1/n^p}-1}{n^{5p}} = \frac{e^{1/n^p}-1}{1/n^p \, n^p n^{5p}} = \frac{e^{1/n^p}-1}{1/n^p \; n^{6p}} \xrightarrow[n\to\infty]{} 0$$ +2

Leibniz $\Rightarrow \sum (-1)^n a_n$ konverguje +1