1. $\int \underbrace{\frac{1}{\sqrt{x^2+x+1}}}_{f}$, $p \in t$ $D(f) = \mathbb{R}$

$\sqrt{x^2+x+1} \overset{(+2)}{=} x + t$ $= \frac{t^2-1}{1-2t} + t = \frac{t^2-1+t-2t^2}{1-2t} \overset{(+1)}{=} \frac{-t^2+t-1}{1-2t}$

$x^2+x+1 = x^2+2tx+t^2$

$x(1-2t) = t^2-1$

$x \overset{(+1)}{=} \frac{t^2-1}{1-2t}$

$dx = \frac{1}{(1-2t)^2}\left(2t(1-2t) - (t^2-1)(-2)\right) \overset{(+2)}{=} \frac{-2(t^2-t+1)}{(1-2t)^2}$

$\rightarrow \overset{(+2)}{\int} \frac{1-2t}{-(t^2-t+1)} \cdot \frac{-2(t^2-t+1)}{(1-2t)^2} = \int \frac{2}{1-2t} \overset{(+1)}{=} -\log|1-2t|$

$\Rightarrow \int f \overset{(+1)}{\underset{c}{=}} -\log|1-2(\sqrt{x^2+x+1}-x)|$, $+ \in \mathbb{R}$

2. $\int e^{-x+y} \overset{+2}{=} \int_1^{\infty} e^{-x}\left(\int_0^{\log x} e^{y}\right) \overset{+1}{=} \int_1^{\infty} e^{-x}\left[e^{y}\right]_0^{\log x}$

$1 \le x < \infty$
$0 \le y \le \log x$

$e^{-x+y} \ge 0$ na M +1

$\overset{+2}{=} \int_1^{\infty} \underset{\underset{-e^{-x}}{\uparrow}}{e^{-x}} \underset{\underset{1}{\downarrow}}{(x-1)} \overset{+2}{=} \underbrace{\left[-e^{-x}(x-1)\right]_1^{\infty}}_{=0} + \int_1^{\infty} e^{-x} \overset{+2}{=} \left[-e^{-x}\right]_1^{\infty} = e^{-1}$

3. $A = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 0 & -2 \\ 2 & 0 & -1 \end{pmatrix} \quad \lambda I - A = \begin{pmatrix} \lambda-1 & 0 & 1 \\ 1 & \lambda & 2 \\ -2 & 0 & \lambda+1 \end{pmatrix} \Rightarrow$

$\det(\lambda I - A) = (-1)^{2+2}\lambda \det\begin{pmatrix} \lambda-1 & 1 \\ -2 & \lambda+1 \end{pmatrix} = \lambda(\lambda^2 - 1 + 2) = \lambda(\lambda^2+1)$

$\sigma(A) = \{0, i, -i\}$ +2

$\lambda = 0$: +2
$\begin{pmatrix} -1 & 0 & 1 \\ 1 & 0 & 2 \\ -2 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow x_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda = i$: +3
$\begin{pmatrix} i-1 & 0 & 1 \\ 1 & i & 2 \\ -2 & 0 & i+1 \end{pmatrix} \sim \begin{pmatrix} 1 & i & 2 \\ i-1 & 0 & 1 \\ -2 & 0 & i+1 \end{pmatrix} \overset{+i}{\sim} \begin{pmatrix} 1 & i & 2 \\ 0 & i+1 & 3-2i \\ 0 & 2i & i+5 \end{pmatrix} \overset{\frac{-2i}{i+1}}{\sim}$

$\sim \begin{pmatrix} 1 & i & 2 \\ 0 & i+1 & 3-2i \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow x_i = \left(-2 - i\frac{(-3+2i)}{1+i}, \frac{-3+2i}{1+i}, 1\right)$

$= \left(\frac{i}{1+i}, \frac{-3+2i}{1+i}, 1\right)$

$\lambda = -i$: +3
$\begin{pmatrix} -i-1 & 0 & 1 \\ 1 & -i & 2 \\ -2 & 0 & -i+1 \end{pmatrix} \sim \begin{pmatrix} 1 & -i & 2 \\ -i-1 & 0 & 1 \\ -2 & 0 & -i+1 \end{pmatrix} \overset{i+1}{\sim} \begin{pmatrix} 1 & -i & 2 \\ 0 & -i+1 & 3+2i \\ 0 & -2i & 5-i \end{pmatrix} \overset{\frac{2i}{1-i}}{\sim}$

$\sim \begin{pmatrix} 1 & -i & 2 \\ 0 & -i+1 & 3+2i \\ 0 & 0 & 0 \end{pmatrix} \quad x_{-i} = \left(-2 + i\frac{3+2i}{-1+i}, \frac{3+2i}{-1+i}, 1\right)$

4. $\lim_{x\to 0} \dfrac{(\cos x)^x - 1 + \frac{x^3}{2}}{x^5}$

- $a^b = e^{b\log a} \Rightarrow (\cos x)^x = e^{x\log(\cos x)} = 1 + x\log\cos x + \dfrac{(x\log\cos x)^2}{2!} + \dots$

- $\log\cos x = \log(1+(\cos x - 1)) = (\cos(x) - 1) - \frac{1}{2}(\cos x - 1)^2 + \frac{1}{3}(\cos x - 1)^3 + \dots$

- $\cos x - 1 = -\frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)$ (+2)

$\Rightarrow \log\cos x = \left(-\frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)\right) - \frac{1}{2}\left(-\frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)\right)^2 + \varphi(\cos x - 1)$, tale

(+2) $\dfrac{\varphi(t)}{t^2} \xrightarrow[t\to 0]{} 0$

$\Rightarrow \dfrac{\varphi(\cos x - 1)}{x^4} = \dfrac{\varphi(\cos x - 1)}{(\cos x - 1)^2}\left(\dfrac{\cos x - 1}{x^2}\right)^2 \xrightarrow[x\to 0]{} 0$

$\Rightarrow \log\cos x = -\frac{x^2}{2} - \frac{x^4}{12} + o(x^4) \Rightarrow x\log\cos x \overset{(+1)}{=} -\frac{x^3}{2} - \frac{x^5}{12} + o(x^5)$

- $e^y = 1 + y + \frac{y^2}{2!} + o(y^2) = 1 + y + \frac{y^2}{2!} + \psi(y)$, tale $\dfrac{\psi(y)}{y^2} \xrightarrow[y\to 0]{} 0$

$\Rightarrow e^{x\log\cos x} = 1 + \left(-\frac{x^3}{2} - \frac{x^5}{12} + o(x^5)\right) + \dfrac{\left(-\frac{x^3}{2} - \frac{x^5}{12} + o(x^5)\right)^2}{2!} + \psi(x\log\cos x)$ (+3)

$\left[\dfrac{\psi(x\log\cos x)}{x^5} = \dfrac{\psi(x\log\cos x)}{x^2(\log\cos x)^2}\,\dfrac{\left(-x^3/2 - x^5/12 + o(x^5)\right)^2}{x^5} \xrightarrow[x\to 0]{} 0\right]$

$= 1 - \frac{x^3}{2} - \frac{x^5}{12} + o(x^5)$

$\Rightarrow \dfrac{(\cos x)^x - 1 + \frac{x^3}{2}}{x^5} \overset{(+2)}{=} -\frac{1}{12} + \frac{o(x^5)}{x^5} \xrightarrow[x\to 0]{} -\frac{1}{12}$

5. $f(x,y) = xy^2 + 2x^2 + y^2 + x$

$f_x = y^2 + 4x + 1$ | $y^2 + 4x + 1 = 0$

$f_y = 2yx + 2y$ | $2y(x+1) = 0 \Rightarrow$ $x = -1 \to y = \sqrt{3}$, $y = -\sqrt{3}$; $y = 0 \to x = -1/4$ +2

podezřelé body jsou $(-1, \sqrt{3}), (-1, -\sqrt{3}), (-\frac{1}{4}, 0)$

$$D^2 f(x,y) = \begin{pmatrix} 4 & 2y \\ 2y & 2x+2 \end{pmatrix}$$ +2

$(-1, \sqrt{3})$: $\begin{pmatrix} 4 & 2\sqrt{3} \\ 2\sqrt{3} & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & 2\sqrt{3} \\ 0 & -3 \end{pmatrix} \sim \begin{pmatrix} 4 & 0 \\ 0 & -3 \end{pmatrix}$ ID ... není extrém

+2

$(-1, -\sqrt{3})$: $\begin{pmatrix} 4 & -2\sqrt{3} \\ -2\sqrt{3} & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & -2\sqrt{3} \\ 0 & -3 \end{pmatrix} \sim \begin{pmatrix} 4 & 0 \\ 0 & -3 \end{pmatrix}$ ID ... není extrém

+2

$(-\frac{1}{4}, 0)$: $\begin{pmatrix} 4 & 0 \\ 0 & \frac{3}{2} \end{pmatrix}$ PD ... lokální minimum

+2