1. $\int \log(x^2+1)\cdot\frac{1}{(x-1)^2} \overset{+3}{=} -\frac{\log(x^2+1)}{x-1} + \int \frac{2x}{(x^2+1)(x-1)} =$

$\downarrow \qquad \uparrow$

$\frac{2x}{x^2+1} \qquad \frac{-1}{x-1}$

$$\frac{2x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

$$2x = (Ax+B)(x-1) + C(x^2+1) \xrightarrow{x=1} C=1$$

$$= x^2(A+C) + x(-A+B) + (-B+C)$$

$\Rightarrow A=-1,\ B=1$

$$\overset{+3}{=} -\frac{\log(x^2+1)}{x-1} + \int\frac{1}{x-1} + \int\frac{1}{x^2+1} - \frac{1}{2}\int\frac{2x}{x^2+1} = \cancel{-\frac{\log(x^2+1)}{x-1} + \log|x-1| + \operatorname{arctg} x - \frac{1}{2}\log(x^2+1)}$$

$$\overset{+4}{\underset{C}{=}} -\frac{\log(x^2+1)}{x-1} + \log|x-1| + \operatorname{arctg} x - \frac{1}{2}\log(x^2+1), \quad x\in(-\infty,1)\cup(1,\infty)$$

2. $f(x,y) = x^2 y \qquad D = \operatorname{co}\{[0,0],[1,0],[1,1],[2,1]\}$

$$\iint_D dx\,dy = \underset{\substack{0\le x\le 1\\ 0\le y\le x}}{\overset{+3}{\int}} x^2y + \underset{\substack{1\le x\le 2\\ x-1\le y\le 1}}{\overset{+3}{\int}} x^2y = \int_0^1 x^2\frac{x^2}{2} + \int_1^2 x^2\frac{(1-(x-1)^2)}{2} =$$

$$= \frac{1}{2}\cdot\frac{1}{5} + \frac{1}{2}\int_1^2 x^2(2x-x^2) = \frac{1}{10} + \frac{1}{2}\,2\left[\frac{x^4}{4}\right]_1^2 - \frac{1}{2}\left[\frac{x^5}{5}\right]_1^2$$

$$\overset{+4}{=} \frac{1}{10} + \frac{1}{4}(2^4-1) - \frac{1}{10}(2^5-1)$$

3. $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 0 & 2 \\ -2 & 2 & 1 \end{pmatrix} \quad \lambda I - A = \begin{pmatrix} \lambda-1 & -2 & 2 \\ 1 & \lambda & -2 \\ 2 & -2 & \lambda-1 \end{pmatrix} \Rightarrow \det(\lambda I - A) =$

$$= \det\begin{pmatrix} 0 & \lambda(1-\lambda)-2 & 2\lambda \\ 1 & \lambda & -2 \\ 0 & -2\lambda-2 & \lambda+3 \end{pmatrix} = (-1)\det\begin{pmatrix} \lambda(1-\lambda)-2 & 2\lambda \\ -2\lambda-2 & \lambda+3 \end{pmatrix} = (\lambda-1)(\lambda+2)(\lambda-3)$$

$$\overset{+4}{\Rightarrow} \sigma(A) = \{1,-2,3\}$$

$\lambda = 1$: $\begin{pmatrix} 0 & -2 & 2 \\ 1 & 1 & -2 \\ 2 & -2 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & -2 & 2 \\ 1 & 1 & -2 \\ 0 & -4 & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & -2 & 2 \\ 1 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

(+2)

$\Rightarrow \begin{matrix} x + y - 2z = 0 \\ -y + z = 0 \end{matrix} \Rightarrow \ker(1 \cdot I - A) = \mathrm{lin}_{\mathbb{C}} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

$\lambda = -2$: $\begin{pmatrix} -3 & -2 & 2 \\ 1 & -2 & -2 \\ 2 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 0 & -8 & -4 \\ 1 & -2 & -2 \\ 0 & 2 & 1 \end{pmatrix}$

(+2)

$\Rightarrow \begin{matrix} x = 2y + 2z \\ 2y + z = 0 \end{matrix} \Rightarrow \ker(-2I - A) = \mathrm{lin}_{\mathbb{C}} \left\{ \begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix} \right\}$

$\lambda = 3$: $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 3 & -2 \\ 2 & -2 & 2 \end{pmatrix} \sim \begin{pmatrix} 0 & -8 & 6 \\ 1 & 3 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

(+2)

$\Rightarrow \begin{matrix} x = 2z - 3y \\ y = \frac{3}{4} z \end{matrix} \Rightarrow \ker(3I - A) = \mathrm{lin}_{\mathbb{C}} \left\{ \begin{pmatrix} -1/4 \\ 3/4 \\ 1 \end{pmatrix} \right\}$

4. $\cos y = 1 - \frac{y^2}{2!} + \frac{y^4}{4!} + o(y^5)$ (+2) $\Rightarrow \cos(\sin x) = 1 - \frac{1}{2!}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)\right)^2 +$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)$ (+2) $\quad + \frac{1}{4!}\left(x - \frac{x^3}{3!} + o(x^4)\right)^4 + \varphi(\sin x) =$

$\left[\frac{\varphi(\sin x)}{x^5} = \frac{\varphi(\sin x)}{(\sin x)^5} \cdot \frac{(\sin x)^5}{x^5} \xrightarrow[x \to 0]{} 0 \right]$

(+4)

$= 1 - \frac{x^2}{2} + x^4 \underbrace{\left(\frac{1}{6} + \frac{1}{24}\right)}_{= \frac{5}{24}} + o(x^5)$

$\Rightarrow \frac{\cos(\sin x) - 1 + \frac{x^2}{2}}{x^4} = \frac{5}{24} + \frac{o(x^5)}{x^4} \xrightarrow[x \to 0]{} \frac{5}{24}$ (+2)

5. $f(x,y,z) = 3y^4 + 3yx^2 - x^3 + z^2 - z$

$f_x = 6yx - 3x^2 \Rightarrow x(6y - 3x) = 0 \Rightarrow$
- $x = 0,\ y = 0,\ z = 1/2$
- $x \neq 0 \Rightarrow x = 2y \Rightarrow 12y^3 + 12y^2 = 0$

$f_y = 12y^3 + 3x^2$

$12y^2(y+1) = 0$

$f_z = 2z - 1$ (+3)

$y = -1$

$x = -2$

$\Rightarrow \overset{\alpha}{[0, 0, 1/2]},\ \overset{\beta}{[-2, -1, 1/2]}$

$$\nabla^2 f(x,y,z) = \begin{pmatrix} 6y - 6x & 6x & 0 \\ 6x & 36y^2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \ (+2)$$

$$\alpha: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$ je semidefinitní. Ale:

$f(x, 0, 1/2) = -x^3 - \frac{1}{4} = -x^3 + f(0,0,1/2)$

(+3)

$\Rightarrow$: $x > 0 \Rightarrow f(x, 0, 1/2) < f(0,0,1/2)$

$x < 0 \Rightarrow f(x, 0, 1/2) > f(0,0,1/2)$

$\Rightarrow$ α je sedlový bod

$$\beta: \begin{pmatrix} 6 & -12 & 0 \\ -12 & 36 & 0 \\ 0 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 6 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$ PD $\Rightarrow$ ostré lokální minimum

(+2)

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