Abstracts

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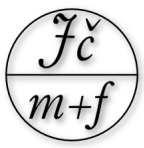

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Invited lectures and tutorials

Automata and logically defined tree languages

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In the theory of automata, one of the leading themes is the connection between languages recognised by automata and languages definable in logic (typically monadic second-oder logic or its fragments). An example is the language $a \star b\star$ which is defined by a formula that says "for every position with label b, every later position also has label b" . For word languages there are a number of theorems which characterise the expressive power of logics in terms of the automata that recognise them. A famous example is the Schutzenbeger-McNaughton-Papert Theorem, which says that a word language is definable in first-order logic if and only if it is recognised by an automaton that is counter-free (a structural condition on automata, which forbids a certain kind of cycles). In my course, I will discuss the automata/logic connection. In particular, I will discus the case of finite trees, where the automata are the same thing as finite algebras in the sense of universal algebra.

Logic, Algebra and Implication

Carles Noguera i Clofent
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The goal of this short course is to give an overview of the general theory of weakly implicative logics, which provides an algebraic approach to logics with a reasonable implication connective (hence, covering the majority of non-classical logics studied in the literature). Namely, we will follow the next plan:

1. We will start by recalling basic syntactical notions and giving a purely syntactical definition of logic system. After testing the definition with three extreme, mostly uninteresting, examples, we immediately give some of most important examples that one can find in the literature: classical logic, intuitionistic logic, Lukasiewicz logic, Gödel-Dummett logic, BCI, and BCK. We introduce a very general complete semantics for the logic of matrices based on arbitrary algebras. In order to obtain a more meaningful semantics, we introduce weakly implicative logics as systems with reasonable implication connective that allows to induce an equivalence relation between formulas and obtain a better

complete semantics of reduced models, where equivalent elements have been already identified. After that, we will focus on logics (including all finitary systems) where the completeness theorem can be further restricted to (finitely) relative subdirectly irreducible models. In particular, in the case of classical logic, this improvement allows to restrict the semantics from the class of all Boolean algebras to just the two-element Boolean algebra. Finally, we focus on the class of algebraically implicative logics, where the relation with the semantics can be strengthened in such a way that, instead of matrices, we can work directly with algebras.

2. We will consider an enriched propositional language that contains more than just the classical connectives, namely an additional implication, an additional conjunction and two additional constants. We give several logical laws that should govern a reasonable minimal behavior of such connectives and define implicitly obtain a very weak substructural logic that we call SL, where the requirements on the implications ensure that it is weakly implicative. After studying several syntactical properties of SL, se use it to define whole family of substructural logics as suitable expansions of (fragments of) SL. We introduce an algebraic semantics for SL and the main substructural logics based on lattice-ordered residuated groupoids and show their relation with well-known logic systems studied in the literature. Then, we present a rather involved syntactical approach to deduction theorem for substructural logics, provided that they can be presented by a suitable proof system (the so-called almost MP-based axiomatizations). This forces us to find such kind of presentations for the main substructural logics. As a consequence of the deduction theorem we obtain a form of proof by cases property (PCP), typically satisfied by the disjunction connective in classical logic, but in general not true for the lattice-disjunction connectives in most substructural logics.

3. The fact that in many substructural logics the PCP is obtained by using a set of formulas instead of a single disjunction connective motives the abstract study of generalized disjunctions (given by sets of formulas). First we will introduce some convenient notation and terminology and several notions and disjunctions and proof by cases yielding a hierarchy of non-classical logics that we will illustrate and separate with suitable examples. Then, we will prove several useful characterizations of the proof by cases properties. We finally show some applications of generalized disjunctions in weakly implicative logics.

4. Finally, we will focus on semilinear logics (those complete w.r.t. linearly ordered models). We will characterize semilinear logics in terms of linear filters, a syntactical metarule, and the coincidence of finitely subdirectly irreducible models and linear models. We show which of the examples of weakly implicative logics considered before are actually semilinear logics and prove which of them are not semilinear with respect to any implication. Then we will study the problem of, given an arbitrary weakly implicative logic (in particular, given a substructural logic), finding its minimal semilinear extension. We will use the presence of disjunction to prove better characterizations of semilinearity

leading to axiomatizations of the minimal semilinear logic over a given logic. Finally, we will focus on refinements of the completeness theorem, by restricting to specific kinds of linearly ordered models (in particular, densely ordered models, algebras over the rationals or the reals numbers, and finite chains).

Identities in tropical matrix semigroups and the bicyclic monoid

Mark Kambites

University of Manchester, Manchester, United Kingdom

I will discuss recent joint work with Laure Daviaud and Marianne Johnson. We show among other things that the variety generated by the bicyclic monoid (or equivalently, by the free monogenic inverse monoid) coincides with that generated by the monoid of 2×2 upper triangular matrices over the real tropical semiring. This result answers a question of Izhakian and Margolis. It allows us to solve the computational problem of checking identities in the bicyclic monoid (which is ostensibly a discrete combinatorial problem) in polynomial time, by reduction to checking equivalence of formal tropical polynomials (which is a continuous linear programming problem). The talk will be aimed at a general audience, with all required background from semigroup theory and tropical algebra explained.

Promise constraint satisfaction

Jakub Opršal

Technische Universität Dresden, Dresden, Germany
Coauthors: J. Bulín, A. Krokhin, and others

The algebraic approach to constraint satisfaction problem (CSP) has been a significant direction in universal algebra in the past few years that culminated in two algebraic proofs of Feder-Vardi dichotomy conjecture, one by A. Bulatov and the other by D. Zhuk. The talk will focus on a new generalization of CSP that was suggested by Austrin, Håstad and Guruswami: promise constraint satisfaction problem (PCSP). The PCSP significantly extends the standard decision CSP, and contains some long-standing open questions, e.g. approximate graph coloring: fix $c \geq k$, given a graph that is promised to be c colorable, find a k-coloring. We show how universal algebra can help to resolve such problems. The recent progress on this topic suggests more interaction between combinatorics, approximation of CSP, and algebra.

Clones and Galois connections

Reinhard Pöschel
Technische Universität Dresden, Dresden, Germany

Galois connections appear "almost everywhere" and turn out to be a useful tool for the investigation of mathematical structures. In this talk several examples are considered in particular in connection with clones of operations and relations where "the most basic Galois connection in algebra" Pol-Inv (induced by the preservation property) comes into play. Some history, basic results and open problems will be sketched.

Contributed talks

Commutativity in the lattice of topologizing filters of a commutative semiartinain ring

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Coauthors: John van den Berg

The set FilR of all right topologizing filters on a fixed but arbitrary ring R admits a monoid operation ':' that is in general noncommutative, even in cases where the ring R is commutative. Earlier results show (see[6]) that commutativity of the monoid operation ':', when imposed as a condition on FilR manifests as a finiteness condition on R. In a quite separate and much earlier study, T. Shores [1974] has shown that if R is a commutative semiartinian ring, then R will be artinian precisely if the first two terms in the Loewy series of RR namely $\mathrm{soc}(RR)$ and $\mathrm{soc}\,2(RR)$ are finitely generated. T.Shores goes further to produce examples which show that the finiteness of just $\mathrm{soc}(RR)$ exercises no constraint whatsoever on the length of the right R-module RR. The main result of this paper asserts that a commutative semiartinian ring R will be artinian precisely if soc(RR) is finitely generated and the monoid operation ':' on FilR is commutative. A family of commutative semiartinian rings of Loewy length 3 is constructed and this used to delineate earlier theory. In particular,and within this family, rings R are exhibited such that $\mathrm{soc}(RR)$ and $\mathrm{soc}\,2(RR)/\,\mathrm{soc}(RR)$ have infinite length, yet the monoid operation ':' is commutative.

Algebraic approaches to the study of the polynomial closure of classes of regular languages

Jana Bartoňová

Masaryk University, Brno, Czechia

A polynomial closure is an operation on classes of regular languages, used for creating concatenation hierarchies of regular languages. The known result by Branco and Pin gives an equational description of the polynomial closure $\mathrm{Pol}(C)$ of a given quotienting lattice C of regular languages. Place and Zeitoun have recently reformulated this equational description in terms of C-separation. We study connections of these two mentioned and other different approaches to the description of the polynomial closure.

On a stronger reconstruction notion for clones

Mike Behrisch
Technische Universität Wien
Coauthors: Edith Vargas-García (ITAM)

We discuss the analogue of reconstruction of automorphism groups up to bi-definability for clones. Rubin's weak $\forall\exists$-interpretations play a key role. For clones over the monoids of self-embeddings of certain countable homogeneous relational structures (including $(\mathbb{Q}, <)$, the random graph, the random digraph and the random tournament) we obtain positive results, for some others we present partial answers.

Structures with small orbit growth

Bertalan Bodor
TU Dresden
Coauthors: Manuel Bodirsky

Let C be the class of those countable structures $\mathfrak{A}$ for which there exist constants c_1 and $c < 1$ so that the number of orbits of $\mathrm{Aut}(\mathfrak{A})$ acting on n-tuples with pairwise distinct entries is at most $c_1 n^{cn}$. We show that this class is also equal to the class of finite coverings of first-order reducts of unary ω-categorical structures. In addition we show that Thomas' conjecture holds for the this class, i.e. every structure in C has finitely many first-order reducts up to interdefinability.

The class C can also be interesting in the study of infinite domain CSPs. M. Bodirsky and A. Mottet showed that the CSP dichotomy conjecture holds for first-order reducts of unary structures (using the finite domain CSP dichotomy theorem by A. A. Bulatov and D. N. Zhuk). This is a proper subclass of C, and it would be interesting to see whether this result can be generalized to the class C, as well.

Joint work with Manuel Bodirsky.

Generating $\oplus$-functions

Martin Broušek
Palacký University Olomouc
Coauthors: Michal Botur

A nondecreasing function $f\colon [0,1] \longrightarrow [0,1]$ is called a $\oplus$-*function* if $f(0) = 0$ and it satisfies subadditivity, i.e., $f(x \oplus y) \leq f(x) \oplus f(y)$. These functions arise as transformations of generalised (fuzzificated) subalgebras and they form a convex set. It is than natural to ask what the extremes (vertices) of this set look like since they can be viewed as generators of all $\oplus$-functons. We also explore possible restrictions of this notion to a finite domain.

Homomorphisms between product of algebras

Ivan Chajda
Palacký University Olomouc
Coauthors: Helmut Länger, Martin Goldstern

Let K be a congruence distributive variety. An algebra A is called hereditary directly irreducible (HDI) if every of its subalgebras is directly irreducible. It is shown that every homomorphism from a finitie direct product of arbitrary algebras from K to an HDI algebra from K is essentially unary. Hence, every homomorphism from a finite direct product of algebras from K to an arbitrary direct product of HDI algebras from K can be expressed as product of homomorphisms from single algebras. A homomorphism from an infinite direct product of algebras from K to an HDI algebra is not in general essentially unary, but is always factored through a suitable ultraproduct. Among lattices, HDI algebras are just the chains.

The variety of reduced Rickart rings

Insa Cremer
University of Latvia, Riga, Latvia

A ring R is called Rickart ring if, for every $a \in R$, there exist idempotents $e, f \in R$ such that for all $x \in R$, $ax = 0$ iff $ex = x$, and $xa = 0$ iff $xf = x$. A ring is called reduced if $a^n = 0$ implies $a = 0$ for every positive integer n. The class of reduced Rickart rings with suitably chosen operations is a variety. In this variety, a reduced Rickart ring is subdirectly irreducible if and only if it is a domain.

Lattices with many congruences are planar

Gábor Czédli
University of Szeged, Szeged, Hungary

I what follows, let L be an n-element finite lattice. We prove that if L has strictly more than 2^{n-5} congruences, then it is planar. This result is sharp for $n \geq 8$. (For $n < 8$, L is automatically planar, no matter what $|\mathrm{Con}(L)|$ is.)

The earlier achievements that paved to way to the present one are the following. R. Freese pointed out that $|\mathrm{Con}(L)| \geq 16 \cdot 2^{n-5}$. The present author noticed that the second largest value of $|\mathrm{Con}(L)|$ is $8 \cdot 2^{n-5}$ (for $n \geq 4$). The third, fourth, and fifth largest values of $|\mathrm{Con}(L)|$ were given by J. Kulin and C. Mureşan. Not only these five largest numbers but the lattices witnessing them were also determined by these authors. Their results imply the planarity of L if $|\mathrm{Con}(L)|$ is at least $\frac{7}{2} \cdot 2^{n-5}$, the fifth largest number. See `https://arxiv.org/abs/1807.08384` for more details.

Special elements in pseudocomplemented effect algebras

Petr Emanovský
Palacký University Olomouc

The talk deals with special elements of the so-called effect algebras, special case of basic algebras which correspond in a natural way to bounded lattices with antitone involutions. We study two groups of the special elements. The first one regarding the underlying lattice of the basic algebra (distributive, standard and neutral elements) and the second one defined for basic algebras (sharp, boolean and central elements). Finally, we investigate effect algebras with pseudocomplementation and relate this property to the mentioned above special elements.

Expansions of $(\mathbb{Z}_p \times \mathbb{Z}_q, +)$

Stefano Fioravanti
Johannes Kepler University Linz

We investigate the expansions of $\mathbb{Z}_p x \mathbb{Z}_q$, for two distinct prime numbers p and q. A $+$-clone is a subset of these functions which contains $+$, the projections and is closed under composition. These clones describe all the expansions of $\mathbb{Z}_p \times \mathbb{Z}_q$. We characterize these expansions using the description of the (p, q)-linear closed clonoids and S. Kreinecker's description of the expansions of $(\mathbb{Z}_p, +)$. Furthermore we prove that the cardinality of these kind of expansions is finite.

This work was supported by the Austrian Science Fund FWF (P29931).

On an index of finitely generated $\mathbb{Z}[G]$-modules

Pavel Francírek
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Let $G = \langle \gamma \rangle$ be a cyclic group of order n. Let S be a finite set of indices. Let M be a finitely generated $\mathbb{Z}[\langle \gamma \rangle]$-module whose generators will be denoted by $\xi_i, i \in S$. For each $i \in S$ let $\varkappa_i$ be an element of M given by $\varkappa_i = H_i(\gamma)\xi_i$ for some $H_i(X) \in \mathbb{Z}[X]$. By N we shall denote the submodule of M generated by all the elements $\varkappa_i$. We further assume that the elements ξ_i and $\varkappa_i$ satisfy certain relations. Our goal is to compute the index $[M\colon N]$.

The balanced $\{2,3\}$-hyperidentities of length four in invertible algebras and $\{3\}$-hyperidentities of associativity in semigroups

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The present paper is devoted to the study of balanced 2, 3-hyperidentities of the length of four in invertible algebras and 3-hyperidentities of as-sociativity in semigroups. The following second order formula is called hyperidentity:

$$\forall X_1, \ldots, X_m \forall x_1, \ldots, x_n (W_1 = W_2), \qquad 1$$

where X_1,...,X_m are the functional variables, and $x_1, \ldots, x_n$ are the object variables in the words (terms) W_1, W_2. Usually, a hyperidentity is specified without universal quantifiers of the prefix of the equality: $W_1 = W_2$. According to the definition, the hyperidentity $W_1 = W_2$ is said to be satisfied in the algebra (Q, Σ) if this equality holds when every functional variable X_i is replaced by any arbitrary operation of the corresponding arity from Σ and every object variable x_j is replaced by any arbitrary element from Q.

If the arities of the functional variables are: $|X_1| = n_1$,...,$|X_m| = n_m$, then the hyperidentity $W_1 = W_2$ is called $\{n_1, \ldots, n_m\}$-hyperidentity.

A hyperidentity is balanced if each object variable of the hyperidentity occurs in both parts of the equality $W_1 = W_2$ only once. A balanced hyperidentity is called first sort hyperidentity, if the object variables on the left and right parts of the equality are ordered identically. The number of the object variables in a balanced hyperidentity is called length of this hyperidentity.

The algebra (Q, Σ) with the binary and ternary operations is called $\{2,3\}$-algebra. A $\{2,3\}$-algebra is called non-trivial, if the sets of its binary and ternary operations are not singleton.

The present paper aims at classifying of the balanced $\{2,3\}$-hyperidentities of length four in invertible algebras and the description of the invertible algebras in which these hyperidentities hold, as well as at the description of the semigroups that polynomially satisfy ternary associative hyperidentities. The following main results will be proved in the talk.

1. The balanced first sort $\{2,3\}$-hyperidentities of length four in non-trivial invertible algebras are classified;

2. The invertible $\{2,3\}$-algebras with a binary group operation, which satisfy the balanced first sort $\{2,3\}$-hyperidentities of the length four are described;

3. The invertible $\{2,3\}$-algebras with a ternary group operation, which satisfy the balanced first sort $\{2,3\}$-hyperidentities of the length four are described;

4. The classes of the semigroups, which polynomially satisfy the associative $\{3\}$-hyperidentities are described.

Semigroups with a completely simple kernel

Roman Gigoń

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We show that an E-inversive semigroup S has a completely simple kernel K if and only if it contains a primitive idempotent (in that case, K is the set-theoretic union of the groups eSe, where e is a primitive idempotent of S). Moreover, some applications of the above theorem will be pointed out.

Distributive lattices with strong endomorphism kernel property as direct sums

Jaroslav Guričan

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The concept of the strong endomorphism kernel property for an universal algebra has been introduced by Blyth, Silva as follows.

Let $\theta \in \mathrm{Con}(A)$ be a congruence on A. We say that a mapping $f\colon A \to A$ is *compatible* with θ if $a \equiv b(\theta)$ implies $f(a) \equiv f(b)(\theta)$. An endomorphism of A is called *strong*, if it is compatible with every congruence $\theta \in \mathrm{Con}(A)$. An algebra A has the *strong endomorphism kernel property* (SEKP) if every nontrivial congruence relation on A is the kernel of a strong endomorphism of A.

Unbounded distributive lattices which have strong endomorphism kernel property (SEKP) were fully characterized using Priestley duality. The characterisation is as follows.

Theorem. *Let L be an unbounded distributive lattice. Then L has SEKP if and only if L is locally finite and there exists $c \in L$ such that for every $x < c$ or $x < c$ intervals $[x, c]$ (if $x < c$) and $[c, x]$ (if $x > c$) are (finite) Boolean.*

We shall call elements c from of this theorem boolean elements of L.

In this note we shall discuss the structure of boolean elements and we show that an unbounded distributive lattice L which has SEKP can be written as a product $L \cong A \times B \times C$, where all A, B, C enjoy SEKP and moreover

A contains exactly one boolean element, B is $\sum((\{0, \mathbf{1}\}, \mathbf{1}); i \in V)) = \{f \in \prod(\{0, 1\},\ i \in V); f(x) \neq \mathbf{1}$ only for finitelly many $x\}$ - a direct sum of V copies of 2 element chain with "distinguished" element 1 (top element) and C is $\sum((\{\mathbf{0}, 1\}, \mathbf{0}); i \in W))$ – a direct sum of W copies of 2 element chain with "distinguished" element 0 (bottom element) for appropriate sets V, W (any of which can be empty).

Moreover, $B \times C$ is isomorphic to a (convex) sublattice consisting of all boolean elements of L.

Two small monoid varieties with the large join

Sergey Gusev
Ural Federal University, Yekaterinburg, Russia

We examine the monoid varieties as algebras of type $(2, 0)$. Recently Marcel Jackson and Edmond Lee exhibited two monoid varieties such that the subvariety lattices of both the varieties are finite, while the subvariety lattice of their join does not satisfy the ascending chain condition. In this work we found monoid varieties $\mathbf{X}$ and $\mathbf{Y}$ such that the subvariety lattices of $\mathbf{X}$ and $\mathbf{Y}$ are finite (in actual fact, $\mathbf{X}$ and $\mathbf{Y}$ contain 7 and 6 subvarieties respectively) and $\mathbf{X} \vee \mathbf{Y}$ covers $\mathbf{X}$ but the subvariety lattice of $\mathbf{X} \vee \mathbf{Y}$ does not satisfy the descending chain condition. Analogous example for semigroup varieties was provided by Mark Sapir in 1991.

Some monounary algebras with EKP

Emília Halušková
MI SAS, Košice, Slovakia

We study monounary algebras $\mathcal{A}$ such that every congruence relation on $\mathcal{A}$ is a kernel of some endomorphism of $\mathcal{A}$. Equivalent conditions with this property for algebras that have finitely many components and algebras that do not have infinitely many non injective components will be shown. Further, we will present a graph description of all algebras that have this property in two classes of monounary algebras. The first is the class of all algebras with injective operation, the other is the class of all algebras, in which finitely many elements enter to every cyclic element.

Pairwise comparable or disjoint elements in a poset

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Coauthors: Sándor Radeleczki

A subset X of a finite lattice L is called CD-independent if the meet of any two incomparable elements of X equals 0. Czédli, Hartmann and Schmidt have an important result about CD-bases (maximal CD-independent subsets) of distributive lattices. In the talk, we define CD-independent subsets in an arbitrary poset in a natural way. Actually, CD-independence is in close relationship with trees. More precisely, if we have a CD-independent subset in a poset and we remove its possible 0, then we obtain a forest. We show that the CD-bases of any poset can be characterized as maximal chains in a related poset. We use this result to investigate CD-bases in semilattices and in some lattice classes. During the talk, I will show several Figures to illustrate the statements.

Homomorphism order of connected monounary algebras

Danica Jakubíková-Studenovská
P.J.Šafárik University, Košice, Slovakia
Coauthors: none

The homomorphism (quasi-)order between algebraic structures A, B of the same type is defined as follows: A is smaller or equal B if there is a homomorphism of A to B. The paper deals with the class of all connected monounary algebras factorized by the equivalence relation corresponding to the relation less or equal; it is denoted L. We prove that L with the homomorphism partial order is a bounded distributive lattice.

The sum of observables on a σ-frame effect algebra

Jiří Janda
Shaanxi Normal University
Coauthors: Yongming Li

The set $\mathcal{BO}(E)$ of bounded observables on a σ-frame effect algebra E forms with respect to Olson order $\leq$ and the sum of observables $+$, which is a natural generalization of the usual point-wise sum of random variables, a lattice ordered semigroup $(\mathcal{BO}(E); +, \leq)$.

We provide a new description of the sum $x + y$ of bounded observables $x, y \in \mathcal{BO}(E)$ and we shows, that the spectrum $\sigma(x+y)$ of $x+y$ behaves similarly as in the case of random variables. We give conditions under which the operation $+$ preserves continuity of spectral resolutions of observables x and y. Moreover, we investigate a notion of a dense observable together with its dual notion of a meager observable.

Meet-irreducibility of congruence lattices of connected algebras

Lucia Janičková
Pavol Jozef Šafárik University, Košice, Slovakia
Coauthors: Danica Jakubíková-Studenovská

It is known that the all congruences of an algebra $\mathcal{A}$ (ordered by inclusion) form an algebraic lattice, denoted Con$\mathcal{A}$. Further, for a given set A, the system of all Con$\mathcal{A}$, where $\mathcal{A}$ is an algebra with the base set A, forms a lattice (with respect to class-theoretical inclusion). We study the meet-irreducibility in this lattice of the congruence lattices, denoted $\mathcal{E}_A$. Since it is known that all meet-irreducible elements in $\mathcal{E}_A$ must be of the form Con(A, f) for a single mapping f, it is sufficient to investigate congruence lattices of monounary algebras. In this talk, we present some results concerning meet-irreducibility of the congruence lattices of connected monounary algebras.

Representations of Boolean lattices by annihilators in associative rings

Małgorzata Jastrzębska
Siedlce University

In this talk R will be an associative ring with $1 \neq 0$. It is well known that the set $\mathcal{I}_l(R)$ of all left ideals in R is complete modular lattice. If $X \subseteq A$ is a subset then let $l(X) = \{r \in R : rX = 0\}$ be the left annihilator of X in R and let $\mathcal{A}_l(R)$ be the set of all left annihilators in R. Then $\mathcal{A}_l(R)$ ordered by inclusion is a lattice but this lattice need not be modular. In this talk we are going to present some results about rings in which $\mathcal{I}_l(R)$ or $\mathcal{A}_l(R)$ is a Boolean lattice.

Convex effect algebras and the Kalmbach monad

Gejza Jenča
Slovak University of Technology Bratislava

In 2004, Harding proved that there is a functor K from the category of bounded posets **BPos** to the category of orthomodular posets **OMP** and that K is left adjoint to the forgetful functor $U\colon \mathbf{BPos} \to \mathbf{OMP}$. Since U and K form an adjoint pair of functors, they induce a monad $T = U \circ K$ on **BPos**. This monad is called the *Kalmbach monad*. In 2015, Jenča proved that the category of effect algebras is equivalent to the Eilenberg-Moore category for the Kalmbach monad.

We prove that there is another monad S on **BPos**, that can be described as a monad arising from the free multiplicative action of the real unit interval $[0, 1]$ on bounded posets. The pair of monads T, S satisfies the distributive laws in the sense of Beck. Therefore, $T \circ S$ is an underlying endofunctor of a monad. We prove that this monad arises from the free-forgetful adjunction between bounded posets and convex effect algebras.

Congruence computations in arithmetical affine complete varieties

Kalle Kaarli
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Coauthors: Alden Pixley

An algebra $\mathbf{A}$ is called arithmetical (affine complete) if its congruences permute and the congruence lattice is distributive (if all finitary congruence compatible functions on $\mathbf{A}$ are polynomials). A variety is called arithmetical (affine complete) if so are all its members. It is well known that a variety is arithmetical iff it admits a Pixley term, i.e.: a ternary term p satisfying the identities $p(x, y, y) = p(x, y, x) = p(y, y, x) = x$. A principal arithmetical variety is a variety V with Pixley term p which determines principal congruences of any $\mathbf{A} \in V$ as follows: $(z, w) \in \mathrm{Cg}(\mathbf{A})$ iff $p(x, y, z) = p(x, y, w)$. Thus, these varieties have equationally defined principal

congruences. The present work is a continuation of our paper of 2006 where we proved that every arithmetical affine complete variety of finite type is principal. Our main results are the following. Let V be an arithmetical affine complete variety of finite type. Then: 1) the finitely generated congruences of any member of V are equationally defined; 2) there are uniform solution formulas for finitely presented systems of pairwise compatible congruences (the Chinese Remainder Theorem); 3) the meet of principal congruences of any member of V is also principal; 4) if a minimal generating algebra of V is regular, then the variety itself is regular and the join of principal congruences of any member of V is again principal.

Classifications of finitely generated semifields and lattice-ordered groups

Vitezslav Kala

Charles University, Prague

I will discuss several recent classification results concerning semifields. Using a theorem of Busaniche-Cabrer-Mundici on lattice-ordered groups, we describe all semifields that are finitely generated as semirings, and in particular show that they have to be additively idempotent (joint work with Miroslav Korbelar). Moreover, we sketch the study of Banach semifields by Eric Leichtnam and outline how it can (hopefully) be extended using the aforementioned results.

Ordered Kovács-Newman semigroups

Jonatan Kolegar

Masaryk University, Brno, Czech Republic

The lattice of pseudovarieties of semigroups is nowadays a widely studied subject and the notion of irreducibility is essential in mathematics in general. Recently, some irreducibility results concerning pseudovarieties were achieved using Kovács-Newman groups and semigroups. We will show the Kovács-Newman technique in ordered case, as the study of pseudovarieties of ordered semigroups is also motivated by formal language theory.

Fuzzy summation of random processes

David Kruml
Masaryk University

Repetitive processes (e. g. those appearing in mass production) display high level of regularity and can be effectively modeled by tree and categorical structures. But in reality, they are biased by a number of random events. A probability of composed events can be guessed by a three-point estimation and properties of central moments. This yields a non-precise but very fast algebraic method for a direct calculation of probability (with no need of repeated simulations) and, as we believe, is still solid for practical applications.

On divisible psBCK-algebras

Jan Kühr
Palacký University Olomouc

I will present some results on the structure of residuation subreducts of divisible integral residuated po-monoids.

On minimality of generating sets of aggregation clones

Zbyněk Kurač
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Coauthors: Radomír Halaš, Jozef Pócs

Aggregation functions on a given bounded poset are those preserving the order and the bounds. Clearly, this class forms a clone, the so-called aggregation clone. The aggregation clone on a finite lattice is known to be finitely generated. The aim of our talk is to discuss several generating sets of this clone with focusing on the minimality of the number of generators. In particular, we will discuss how the number of specific generators depends on a considered lattice.

A simple identity forcing a lattice to be Boolean

Helmut Länger
TU Wien, Vienna, Austria, and Palacký University Olomouc
Coauthors: Léonard Kwuida

We present a simple proof for the fact that a non-empty lattice $(L, \vee, \wedge, ')$ with a unary operation $'$ is a Boolean algebra if and only if it satisfies the identity $(x \wedge y) \vee (x \wedge y') \approx (x \vee y) \wedge (x \vee y')$.

Minimal study of binary minimal clones

Hajime Machida
Hitotsubashi University

A *binary minimal clone* is a term temporarily used to mean a minimal clone generated by a binary idempotent function. For a function f the *pr-distance* of f is tentatively defined to indicate the distance of f from the projections. B. Csákány (1983) determined all minimal clones on a three-element set. There are 48 binary minimal clones on a three-element set.

It turns out that the generators of those binary minimal clones can be classified into three classes: Commutative functions (12), non-commutative functions with the pr-distance 1 (12) and non-commutative functions with the pr-distance 2 (24). (The number in the parenthesis gives the number of minimal clones in each class.)

An attempt is made to generalize some of these results from a three-element set to a finite set with more than 3 elements.

Lattice Congruences Preserving Involutions

Claudia Muresan
University of Cagliari

In [3], we study the congruence lattices of lattices with involutions (in brief, i–lattices); we determine their atoms, inferring characterizations for subdirect irreducibility, prove that they are Boolean when the lattices of lattice congruences are Boolean, prove that the lattice congruences that are comparable to all lattice congruences are full congruences, use our characterization for the full congruences to prove that the variety of distributive i–lattices has the congruence extension property. Finally, we prove that, for any $n \in \mathbb{N}^*$, the n–element i–lattices with the most congruences are exactly the ones that, according to [1], have either the largest or the second largest possible number of lattice congruences, namely, in view of their self–duality, the n–element chain and, for $n \geq 4$ and even, the ordinal sum of the $(n/2-1)$–element chain with the four–element Boolean algebra and another copy of the $(n/2-1)$–element chain, with the involution on the copy of the four–element Boolean algebra coinciding to the Boolean complement, both of which are Kleene algebras; their number of full congruences is $2^{\lfloor n/2 \rfloor}$ and their congruence lattices are, of course, Boolean, with $\lfloor n/2 \rfloor$ atoms; if, on the second n–element lattice mentioned above, we give the involution the other possible definition, which turns the copy of the four–element Boolean algebra into the horizontal sum of two copies of the three–element i–chain, then we obtain half as many congruences and the Boolean lattice with $\lfloor n/2 \rfloor - 1$ atoms as the congruence lattice. Despite what this result suggests, there exist pairs of n–element i–lattices such that one of them has strictly more full congruences, but strictly less lattice congruences than the other; for instance, if $n \geq 6$ and is even, then one of the n–element i–lattices which, according to [2], has the fourth largest possible number of lattice congruences has strictly less full congruences than one of the n–element i–lattices with the fifth largest possible

number of lattice congruences, determined in [2], as well; moreover, the first of these two i–lattices is a Kleene algebra and the second is a pseudo–Kleene algebra. The results mentioned above have as consequences analogous ones for finite Brouwer–Zadeh lattices with the 0 meet–irreducible, which are thus antiortholattices.

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Graph of walks in universal algebra

Miroslav Olšák

Charles University, Prague, Czechia

A graph of walks is a directed graph consisted of all walks of a given length on a given (directed) graph. We give a brief outline of two examples where graphs like these help to solve certain problems in universal algebra. In the first example, some variants of graphs of walks appear as the intermediate steps in proving that all strongly connected loop conditions (one-equation Maltsev conditions) with algebraic length one are equivalent. Second example shows a "local" loop lemma – that an infinite directed graph compatible with an idempotent operation must have a loop even under quite weak algebraic assumptions.

Triples and quadruples in bounded Hilbert algebras and bounded relatively pseudocomplemented posets

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Coauthors: Ivan Chajda, Helmut Länger

In our lecture we introduce quadruples consisting of a Boolean algebra B, a Hilbert algebra D, a certain compatibility relation C between B and D and a join-preserving mapping $\varphi : B \to NucD$ that preserves both the least and the greatest element ($NucD$ is a v-semilattice of closure endomorphisms on D). Our goal is to characterize every bounded Hilbert algebra and relatively pseudocomplemented poset by means of a quadruple. It is worth noticing that characterizing triples were introduced by C. C. Chen and G.Grätzer and intensively studied by T. Katriňák and

his collaborators. In our general case, triples do not produce a representation and hence we must use the so-called characterizing quadruples. Also, our results on characterizing Hilbert quadruples should be compared with the results by J. Cirulis on quasi-decompositions.

In our construction we follow the idea of W. Nemitz for constructing bounded implicative semilattices having a given Boolean algebra for closed algebra, and a given implicative semilattice for dense filter. As Nemitz we work with an action of the Boolean algebra on dense elements but instead of factorizing the cartesian product of the Boolean algebra with the dense filter we directly specify possible pairs of elements of our Boolean algebra and dense elements.

Left distributive biracks

Agata Pilitowska

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Coauthors: Přemysl Jedlička, Anna Zamojska-Dzienio

An algebra $(X, \circ, \backslash_\circ, \bullet, /_\bullet)$ with four binary operations is called a *birack*, if $(X, \circ, \backslash_\circ)$ is a left quasigroup, $(X, \bullet, /_\bullet)$ is a right quasigroup and the following identities hold for any $x, y, z \in X$:

$$x \circ (y \circ z) = (x \circ y) \circ ((x \bullet y) \circ z),$$

$$(x \circ y) \bullet ((x \bullet y) \circ z) = (x \bullet (y \circ z)) \circ (y \bullet z),$$

$$(x \bullet y) \bullet z = (x \bullet (y \circ z)) \bullet (y \bullet z).$$

Biracks appear in low dimensional topology. They also provide set-theoretical solutions to the famous Yang-Baxter equation.

A birack is called *left distributive* if it additionally satisfies:

$$x \circ (y \circ z) = (x \circ y) \circ (x \circ z).$$

During the talk we present the structure of all left distributive biracks and show their importance in construction of all multipermutation solutions of level 2 of the Yang-Baxter equation.

Congruence FD-maximal algebras

M. Ploščica

Slovak Academy of Sciences Košice

We consider the problem of describing the congruence lattices of finite algebras in congruence-distributive varieties. We concentrate on the following special case.

A variety $\mathcal{V}$ is called *congruence FD-maximal*, if for every finite distributive lattice L the following two conditions are equivalent:

(i) L is isomorphic to $\operatorname{Con} B$ for some $B \in \mathcal{V}$;

(ii) for every meet-irreducible $x \in L$, the lattice $\uparrow x$ is isomorphic to $\operatorname{Con} T$ for some (subdirectly irreducible) $T \in \mathcal{V}$.

(Notice that (i) always implies (ii).)

The concept of congruence FD-maximality can also be considered for individual algebras, in the following sense: A is congruence FD-maximal iff the class of all finite members of $\operatorname{Con} \mathrm{P}_s \mathrm{H}(A)$ is as large as possible by the necessary condition.

The study of congruence FD-maximal algebras is an essential part of the study of congruence FD-maximal varieties. We consider some special types of congruence distributive varieties and present a criterion for them, characterizing the congruence FD-maximality.

On the lattice of complete join-endomorphisms of a complete lattice

Sándor Radeleczki

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Coauthors: Kalle Kaarli

It was proved by G. Grätzer and E.T. Schmidt that the complete join-endomorphisms of a complete lattice L also form a complete lattice. In the case of a lattice L with finite bounded chains, they also showed that this lattice $\operatorname{End}(L, \vee)$ is semimodular if and only if L is distributive. We also have shown that the complete endomorphisms of L form a quantale with respect to the composition and the pointwise join of the endomorphisms. In this talk, we try to describe the completely meet-irreducible, and the completely join-irreducible elements of the lattice $\operatorname{End}(L, \vee)$, and we prove that some properties of the lattice L are "lifted" to $\operatorname{End}(L, \vee)$.

On the realization of refinement monoids by von Neuman regular rings

Pavel Růžička

Charles University in Prague

We outline the realization problem for von Neumann regular rings. The problem asks which refinement monoids appear as the monoids of isomorphism classes of finitely generated projective monoids over von Neumann regular rings. We develop elementary methods of computing these monoids for directly-finite regular rings and we realize some directly finite non-cancellative refinement monoids.

Invertible binary algebras isotopic to an abelian group

Davit Shahnazaryan

Yerevan State University
Coauthors: Sergey Davidov

A binary groupoid $Q(A)$ is a non-empty set Q together with a binary operation A. Binary groupoid $Q(A)$ is called quasigroup if for all ordered pairs $(a, b) \in Q^2$ exists unique solutions $x, y \in Q$ of the equations $A(a, x) = b$ and $A(y, a) = b$. A binary algebra $(Q; \Sigma)$ is called invertible algebra or system of quasigroups if each operation in Σ is a quasigroup operation.

We obtained characterizations of invertible algebras isotopic to an abelian group by the second-order formula.

The Independence of Axioms of Hypergroup over Group

Navasardyan Shant

Yerevan State University

The concept of hypergroup over group arises when one tries to extend the concept of quotient group in case of any subgroup of the given group. This concept generalizes and unifies the concepts of the group, of the field and of the linear space over field. Let H be an arbitrary group. A right hypergroup over group H is a set M together with four mappings which satisfy eight axioms. In our work we showed the independence of each of these axioms and proved theorem which claims that in some special cases four of these axioms can be deduced from the other four.

Cancellable varieties of semigroups

Viacheslav Shaprynskii

Ural Federal University

An element x of a lattice L is *cancellable* if the following holds:

$$\forall y, z \in L \quad x \vee y = x \vee z \;\&\; x \wedge y = x \wedge z \to y = z.$$

A variety of semigroups is *cancellable* if it is a cancellable element of the lattice of all semigroup varieties. Cancellable varieties have been studied in a series of papers by B. Vernikov, D. Skokov, and S. Gusev. In my report I present a complete description of all cancellable varieties.

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On special elements of the lattice of epigroup varieties

Dmitry Skokov

Ural Federal University, Yekaterinburg, Russia

A semigroup S is called an *epigroup* if for any element x of S there is a natural n such that x^n is a *group element* (this means that x^n lies in a some subgroup of S). On an epigroup, a natural unary operation named *pseudoinversion* may be found. This allows us to consider varieties of epigroups as algebras with the operations of multiplication and pseudoinversion.

We continue an examination of special elements of the lattice of epigroup varieties. In this talk I am going to summarize recent works and present some new results.

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Testing is the mother of all knowledge

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Coauthors: Anna Muzsnay, Janka Szeibert, Csilla Zámbó

This is not an algebra talk. This is about teaching algebra (if permitted by the organizers).

This talk is about a new (2007) observation of experintal psychologists on how the memory and the learning procedure works.

Retrieving information from memory after an initial learning phase enhances long-term retention more than restudying the material; an advantage referred to as the (retrieval practice) testing effect. The testing effect has been demonstrated with a variety of practice tests, materials, and age groups.

Test-enhanced learning is a method which uses active recall of the information during the learning process. It has been proved to be efficient concerning learning texts or foreign words, but these experiments were principally carried out in laboratory-environment on psychologist undergraduate students. The topic of this presentation is an experiment on the efficiency of test-enhanced learning used for teaching geometry in a secondary school and Number Theory at the university.. The experiments were carried out in a real secondary school and a real university with real students on real maths lessons. For example, the subjects of our university experiments were six groups of undergraduate pre-service mathematic teachers. Three groups of the six were learning Number Theory using the testing effect and the other three groups were learning by the original way. The experimental and the control groups were statistically indistinguishable and were learning exactly the same topic also they had exactly the same teacher at the lecture. The experimental groups and the control groups learned the same concepts and wrote the same midterm and final test. We compared the results of the two types of group on the midterm- and final test and the outcome is very impressive. The experiment was repeated in teaching complex numbers and polynomials.

Knot-theoretic ternary groups

Anna Zamojska-Dzienio
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Coauthors: Maciej Niebrzydowski, Agata Pilitowska

Knot-theoretic ternary groups, lie in the intersection of two classical areas: ternary group theory and knot theory. Knot theory studies embeddings of simple closed curves in a three-dimensional space $\mathbb{R}^3$. These curves (*knots and links*) are analyzed using *diagrams*, that is, projections on the plane that involve double points. Two link diagrams represent the same link if and only if one can be obtained from the other by a finite sequence of *Reidemeister moves* of type I, II and III, and planar isotopy. One of the main goals of knot theory is to find strong and at the same time computable link invariants. A well-known approach to this problem is to use

assignments of elements of some binary groupoid $(X, *)$ to the arcs of a given link diagram (*colorings*). In this way one obtains the axioms for racks and quandles.

But, one can color the regions of the complement of the diagram using a ternary algebra $(A, [\,])$. In this talk we describe various properties of ternary groups which appear in this way. Using special attributes of such ternary groups one can construct a ternary invariant of unoriented flat virtual links.
