A CONSTRUCTION OF BIG MAXIMAL COHEN MACAULAY MODULES

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Let R be a ring, and let M_R be a right R-module. Denote by $V(M)$ the monoid of isomorphism classed of direct summands of M^n for some $n \in \mathbb{N}$, and by $V^*(M)$ the monoid of isomorphism classed of direct summands of $M^{(\aleph_0)}$.

Let M be a module with a semilocal endomorphism ring. Then $V(M)$ can be seen, in a natural way, as a submonoid of $\mathbb{N}^k$; moreover, the precise class of monoids that appears in this way is the one of finitely generated reduced Krull monoids [1] and the same is true when we restrict to M to be a finitely generated module over a local commutative noetherian ring [4] (see also [3]).

In [2], it was provad that if R is a noetherian semilocal ring, then $V^*(R)$ can be identified with the set of solutions in $(\mathbb{N} \cup \{\infty\})^k$ of a system of congruences and linear equations of the form

$$D \cdot T \in \begin{pmatrix} m_1(\mathbb{N} \cup \{\infty\}) \\ \vdots \\ m_n(\mathbb{N} \cup \{\infty\}) \end{pmatrix} \qquad \text{and} \qquad E_1 \cdot T = E_2 \cdot T$$

where $T = (t_1, \dots, t_k)^t$ and D, E_1, E_2 are matrices with entries in $\mathbb{N}$, and $m_1, \dots, m_n \in \mathbb{N}$.

We point out that reduced Krull monoids is the precise class of monoids formed by the solutions of such systems in $\mathbb{N}^k$.

The main aim of this talk will be to explain how we can apply the above result to compute $V^*(M)$ for modules over noetherian local domains of Krull dimension 1 like the ones that were constructed in [4, 3]. This produces examples of balanced Big Maximal Cohen-Macaulay modules. Moreover, we will allso see that for one dimensional domains of finite Cohen Macaulay type the usual situation is to have such Big Maximal Cohen-Macaulay modules.

The results presented are part of an ongoing long joint project with Pavel Příhoda and Roger Wiegand.

References

[1] A. Facchini, D. Herbera, K_0 *of a semilocal ring*, J. Algebra **225** (2000), 47–69.

[2] D. Herbera and P. Příhoda, *Big Projective Modules over Noetherian Semilocal Rings, J. Reine und Angew. Math.* **648** (2010), 111–148.

[3] G. L. Leuschke and R. Wiegand, Cohen-Macaulay Representations. Mathematical Surveys and Monographs **181**, American Mathematica Society 2012.

[4] R. Wiegand, *Direct-Sum Decompositions over Local Rings*. J. of Alg. **240** (2001), 83–97.

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