

1-tilting over commutative rings

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Abstract: The finite type theorem for tilting classes ([1],[2]) allowed classification of tilting classes and modules various classes of rings, including Dedekind domains ([4]), Prüfer domains ([3]), and almost perfect domains ([5]). Recently, a surprisingly strong result was proved in [6], where all n -tilting classes over a commutative noetherian were described in terms of subsets of the Zariski spectrum, even though there is no hope to classify finitely presented modules in general. We continue in this direction and show that 1-tilting classes over an arbitrary commutative ring are in 1-1 correspondence with faithful Thomason subsets of the spectrum. In particular, we show that a 1-cotilting class over a commutative ring is of cofinite type if and only if it is closed under injective envelopes (which in general fails for non-commutative rings). We also introduce a generalization of Fuchs-Salce tilting module, describing equivalence classes of 1-tilting modules in the commutative setting. In the end, we discuss connections to localization theory, and show exactly when a 1-tilting module arises from a flat ring epimorphism.

References

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