The realization problem for von Neumann regular rings

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Abstract: The realization problem for von Neumann regular rings asks whether any countable conical refinement monoid is isomorphic to the monoid $V(R)$ of isomorphism classes of finitely generated projective modules over some von Neumann regular ring R, see [**?**] for a survey on this problem. We say that a conical refinement monoid M is *tame* in case it can be obtained as a direct limit of finitely generated conical refinement monoids. Otherwise, we say that M is a *wild refinement monoid*, see [**?**]. I have conjectured that all finitely generated conical refinement monoids can be realized by a von Neumann regular K-algebra, for an arbitrary field K. We will describe some partial results in this direction. I will also state a recent monoid-theoretic result [**?**] which gives a detailed description of the class of finitely generated conical refinement monoids in terms of certain partial orders of semigroups, which can be useful for the solution of the conjecture.

In contrast, very little is know about the case of wild refinement monoids. In a recent paper with Ken Goodearl [**?**], we have shown various remarkable differences with the case of tame monoids, for instance we have described a wild conical refinement monoid which can be realized by a von Neumann regular K-algebra for any *countable* field K, but cannot be realized by any von Neumann regular ring which is an algebra over an uncountable field.

References

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