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Abstract: A homomorphism $\pi : G \rightarrow H$ of abelian groups is a *cellular cover* over H , if every homomorphism $\varphi : G \rightarrow H$ lifts to a unique endomorphism $\tilde{\varphi}$ of G such that $\pi\tilde{\varphi} = \varphi$. In case that $\pi : G \rightarrow H$ is an epimorphism we say that

$$0 \rightarrow K \rightarrow G \rightarrow H \rightarrow 0$$

is a *cellular exact sequence*, where K is the kernel of π . Many realisation theorems for groups K and H as the kernel respectively the cokernel of a cellular exact sequence are known. However, two problematic cases remained open: The case of H being of finite rank and the case of H being \aleph_1 -free.

In this paper we first show that for every natural number n and every countable reduced cotorsion-free group K there is a short exact sequence

$$0 \rightarrow K \rightarrow G \rightarrow H \rightarrow 0$$

such that the map $G \rightarrow H$ is a cellular cover over H and the rank of H is exactly n . In particular, the free abelian group of infinite countable rank is the kernel of a cellular exact sequence of co-rank 2 which answers an open problem from Rodríguez–Strüingmann [2].

Moreover, we give a new method to construct cellular exact sequences with prescribed torsion free kernels and cokernels. In particular, we apply this method to the class of \aleph_1 -free abelian groups in order to complement results from Rodríguez–Strüingmann [2] and Göbel–Rodríguez–Strüingmann [1].

References

- [1] R. Göbel, J. L. Rodríguez, and L. Strüingmann , *Cellular covers of cotorsion-free modules*, Fund. Math. 217 (2012), 211-231.
- [2] J. L. Rodríguez and L. Strüingmann, *On cellular covers with free kernels*, Mediterr. J. Math. **6** (2010), 139150, DOI 10.1007/s00009-010-0109-1.