Faith's problem on R-projectivity is undecidable

Jan Trlifaj

Contact: trlifaj@karlin.mff.cuni.cz, MFF UK

Abstract: In Algebra II, Faith asked for what rings R does R-projectivity imply projectivity for each right R-module, that is, when does the dual of Baers Criterion for Injectivity hold true in ModR. Such rings R were called right testing. Sandomierski proved that if R is right perfect, then R is right testing. Puninski has recently noticed that an older result of Trlifaj implies that the converse implication is consistent with ZFC (the proof used Shelahs Uniformization). Moreover, Puninski et al. have shown for a large number of non-right perfect rings R that R is not right testing in ZFC.

Here we prove that the existence of right testing, but not right perfect rings is also consistent with ZFC (the proof uses Jensens Diamond). Thus the answer to the Faiths question above is undecidable in ZFC.