

Modules whose certain submodules are essentially embedded in direct summands

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Abstract: It is well known that if the ring has acc on essential right ideals then for every quasi-continuous module over the ring, the finite exchange property implies the full exchange property. In this paper, we obtain the former implication for the generalizations of quasi-continuous modules over a ring with acc on right annihilators of elements of the module. Moreover, we focus on direct sums and direct summands of weak C_{12} modules i.e., modules with the property that every semisimple submodule can be essentially embedded in a direct summand. To this end, we prove that being weak C_{12} is closed under direct sums. Amongst other results, we provide several counter examples including the tangent bundle of a real sphere of odd dimension over its coordinate ring for the open problem whether weak C_{12} implies C_{12} condition.

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