Domácí úlohy na Binární systémy 2018/19

Ke splnění části zkoušky týkající se pologrup je potřeba alespoň 22 bodů. Úlohy odevzdávejte přednášejícímu v libovolné formě.

1. (8 bodů) Determine the $\mathcal{L}, \mathcal{R}, \mathcal{D}, \mathcal{J}$ relations in (a) the matrix semigroup $M_n(F)$, (b) the symmetric inverse semigroup I_X.

2. (4 body) Show that in the matrix semigroup

$$\{\left(\begin{smallmatrix} a & 0 \\ b & 1 \end{smallmatrix}\right) : \ a, b \in \mathbb{R}, \ a, b > 0\}$$

we have $id = \mathcal{D} \neq \mathcal{J}$.

3. (4 body) Show that a semigroup is a group if and only if it contains no proper left ideals and no proper right ideals (*proper* means strictly smaller than S). Hint: use Green's theorem.

4. (6 bodů) Show that the following are equivalent for a regular semigroup:

(1) it has exactly one idempotent;
(2) it is cancellative (i.e., $xu = xv$ implies $u = v$, and $ux = vx$ implies $u = v$);
(3) it is a group.

5. (6 bodů) Determine whether a) PT_X (partial transformations), b) B_X (binary relations), c) $M_n(F)$ ($n \times n$ matrices over F) are regular or inverse semigroups.

6. (4 body) Consider a $\mathcal{D}$-block D in an inverse semigroup. Show that the number of $\mathcal{L}$-blocks in D equals the number of $\mathcal{R}$-blocks in D. Hint: $L_a \to R_{a'}$ is a bijection.

7. (4 body) Show that $(S, \cdot, ')$ is an inverse semigroup iff for every $x, y, z \in S$

$$x(yz) = (xy)z, \ xx'x = x, \ x'xx' = x', \ xx'yy' = yy'xx'.$$

We did ($\Rightarrow$) at the lecture. For ($\Leftarrow$), the hint is to prove that $e = ee'$ for every e idempotent.

8. (6 bodů) [for students also attending Universal Algebra II] Show that $t(x, y, z, u, v) = xy'zu'v$ is a Taylor term for the variety of inverse semigroups. Show that semigroups are not congruence modular (hint: find a semilattice violating the property) and not meet-semidistributive (hint: find a group violating the property). Show that inverse semigroups have a weak difference term.