

Řetězový zlomek pro π

$$\begin{aligned}\pi &= 3.1415926535897932384626433832795028841971693993751058209749445923\dots \\ &= [3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1, 84, 2, 1, 1, 15, 3, 13, 1, \dots]\end{aligned}$$

$$\begin{aligned}\xi_0 &= \pi \\ a_0 &= [\xi_0] = 3\end{aligned}$$

$$\begin{aligned}\xi_1 &= \frac{1}{\xi_0 - a_0} = \frac{1}{0.141592653589793238462643383279\dots} = 7.06251330593104576979300515257\dots \\ a_1 &= [\xi_1] = 7 \\ a_0 + \frac{1}{a_1} &= \frac{22}{7} = \mathbf{3.1428571428571428571428571428571428571428571428571428571428\dots} \\ |\pi - [a_0, a_1]| &\approx \frac{1}{790} < \frac{1}{7^2}\end{aligned}$$

$$\begin{aligned}\xi_2 &= \frac{1}{\xi_1 - a_1} = \frac{1}{0.062513305931045769793005152570\dots} = 15.99659440668571988892306040475\dots \\ a_2 &= [\xi_2] = 15 \\ a_0 + \frac{1}{a_1 + \frac{1}{a_2}} &= \frac{333}{106} = \mathbf{3.1415094339622641509433962264150943396226415094339622641509433962\dots} \\ |\pi - [a_0, a_1, a_2]| &\approx \frac{1}{12016} < \frac{1}{106^2}\end{aligned}$$

$$\begin{aligned}\xi_3 &= \frac{1}{\xi_2 - a_2} = \frac{1}{0.996594406685719888923060404755\dots} = 1.00341723101337260346414717528\dots \\ a_3 &= [\xi_3] = 1 \\ a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3}}} &= \frac{355}{113} = \mathbf{3.1415929203539823008849557522123893805309734513274336283185840707\dots} \\ |\pi - [a_0, a_1, a_2, a_3]| &\approx \frac{1}{37486291} < \frac{1}{113^2}\end{aligned}$$

$$\begin{aligned}103993/33102 &= \mathbf{3.1415926530119026040722614947737296840070086399613316415926530119} \\ |\pi - [a_0, a_1, a_2, a_3, a_4]| &\approx \frac{1}{1.73 \cdot 10^9} < \frac{1}{33102^2}\end{aligned}$$

$$\begin{aligned}104348/33215 &= \mathbf{3.1415926539214210447087159415926539214210447087159415926539214210} \\ |\pi - [a_0, a_1, a_2, a_3, a_4, a_5]| &\approx \frac{1}{3.31 \cdot 10^{10}} < \frac{1}{33215^2}\end{aligned}$$

...

Všimněte si:

- $[a_0, \dots, a_k]$ se přibližuje k π
- $[a_0, \dots, a_{2k}] < \pi < [a_0, \dots, a_{2k+1}]$
- $|\pi - [a_0, \dots, a_k]| < 1/q^2$, kde q je jmenovatel