$3^{5^{7^9}} \equiv 0 \pmod 3$

$\equiv ? \pmod 4$

... $NSD(3,4) = 1 \Rightarrow$ dle Eulerovy věty stačí určit $5^{7^9} \bmod \underbrace{\varphi(4)}_{=6}$:

$5^{7^9} \equiv (-1)^{7^9} \equiv -1 \equiv 5 \pmod 6$

$\Rightarrow 3^{5^{7^9}} \equiv 3^5 = 9 \cdot 9 \cdot 3 \equiv 2 \cdot 2 \cdot 3 \equiv$ [redacted]

$\equiv 5 \pmod 4$

ČÍLL: $\equiv 0 \pmod 3$

$\equiv 5 \pmod 4$

$\Rightarrow \equiv 12 \pmod{21}$

Euklidův algoritmus

$18 - 6\sqrt{-2}$

$1 + 4\sqrt{-2}$

$\frac{18-6\sqrt{-2}}{1+4\sqrt{-2}} \cdot \frac{1-4\sqrt{-2}}{1-4\sqrt{-2}} = \frac{-30-78\sqrt{-2}}{33}$ [redacted]

$\doteq -1 - 2\sqrt{-2}$

$3 \leftarrow$ zbytek $(18-6\sqrt{-2}) - (-1-2\sqrt{-2})(1-4\sqrt{-2}) = 3$

$1+\sqrt{-2} \leftarrow \frac{1+4\sqrt{-2}}{3} \doteq 0 + 1\cdot\sqrt{-2}$, zbytek $1+\sqrt{-2}$

$0 \leftarrow \frac{3}{1+\sqrt{-2}} = 1-\sqrt{-2}$, zbytek 0

$\Rightarrow NSD = 1+\sqrt{-2}$

$f = 6 \cdot (x^4 - x^2 - 2)$

$c(f)$ $\quad$ $pp(f)$

$6 = 3 \cdot (\sqrt{2})^2 \quad$ v $\mathbb{Z}[\sqrt{2}]$

$v(\sqrt{2}) = 2 \rightsquigarrow \sqrt{2}$ je ireducibilní

$v(3) = 9$, ale prvky normy 3 v $\mathbb{Z}[\sqrt{2}]$ neexistují $\rightsquigarrow$ 3 je ireducibilní

$v(a + b\sqrt{2}) = |a^2 - 2b^2| = 3$ nemá řešení

protože $\equiv 2 \pmod 3$ dělitelné 9

$x^4 - x^2 - 2 = (x^2 - 2)(x^2 + 1)$

$= (x - \sqrt{2})(x + \sqrt{2})(x^2 + 1)$

ired., protože st. 1 $\quad$ ired., protože st. 2 & nemá kořen

$$3 \cdot (\sqrt{2})^2 \cdot (x - \sqrt{2})(x + \sqrt{2})(x^2 + 1)$$

Dle věty o interpolaci hledám polynom stupně ≤ 2.

$f = ax^2 + bx + c$

$\rightsquigarrow$ soustava lineárních rovnic pro a, b, c:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & \alpha \\ \alpha^2 & \alpha & 1 & \alpha+1 \\ (\alpha+1)^2 & \alpha+1 & 1 & 0 \end{array}\right)$$

$\alpha^2 = \alpha + 1$
$(\alpha+1)^2 = \alpha$
$\alpha \cdot (\alpha+1) = 1$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & \alpha \\ 0 & 1 & \alpha & \alpha \\ 0 & 1 & \alpha+1 & \alpha+1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & \alpha \\ 0 & 1 & \alpha & \alpha \\ 0 & 0 & 1 & 1 \end{array}\right) \quad \ldots$$

$c = 1$
$b = 0$
$a = \alpha + 1$

$$(\alpha+1)x^2 + 1$$