Universal Algebra 1 - Homework 4

Deadline 21.12.2021, 17:20

1. Let $\mathcal{V}$ be the variety of algebras $(A, \cdot)$ satisfying the identities

$$x \cdot x \approx x \text{ and } (x \cdot y) \cdot z \approx (z \cdot y) \cdot x.$$

   (a) Show that every member of $\mathcal{V}$ also satisfies the identities

$$\begin{aligned}(x \cdot y) \cdot (z \cdot w) &\approx (x \cdot z) \cdot (y \cdot w)\\ x \cdot (y \cdot z) &\approx (x \cdot y) \cdot (x \cdot z)\\ (y \cdot z) \cdot x &\approx (y \cdot x) \cdot (z \cdot x)\\ y \cdot (x \cdot y) &\approx (y \cdot x) \cdot y\\ (y \cdot x) \cdot x &\approx x \cdot y\end{aligned}$$

   (b) Let $\mathcal{W}$ be the subvariety of $\mathcal{V}$ defined by the additional identity $y \cdot (x \cdot y) \approx x$. Determine $\mathbf{F}_{\mathcal{W}}(x, y)$ (multiplication table).