

Algebraic Invariants in Knot Theory

Practicals 11

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Exercise 1 (10.3.1 rev). Find a braids (with the fewest possible strings) whose closure is the equivalent to the following knots.



Exercise 2 (10.3.4). Suppose that σ_1 is a generator of the braid group B_2 . Prove that, for every m, n integers, $\sigma_1^m = \sigma_1^n$ if and only if $m = n$.

Exercise 3. Prove that

- (i) The braid group B_1 is the trivial group.
- (ii) The braid group B_2 is isomorphic to $(\mathbb{Z}, +, -, 0)$.
- (iii) For $n \geq 1$ the braid group B_n embeds into B_{n+1} .
- (iv) For $n \geq 3$, the braid group B_n is non-abelian, torsion-free, and it contains a subgroup isomorphic to the free abelian group on $\lceil \frac{n}{2} \rceil$ generators.