Algebraic Invariants in Knot Theory
Practicals 8

22 November 2022, Prague

Let A be a symmetric matrix (think of it as the matrix of a quadratic form). We denote its signature by $(z(A), p(A), n(A))$, where $z(A)$ the number of zeros, and $p(A), n(A)$ are the number of positive / negative values, in its diagonal form. Let $\sigma(A) = p(A) - n(A)$.

For a link K, we define its *signature* by $\sigma_K = \sigma(M_K + M_K^T)$, and *nullity* by $n_K = z(M_K + M_K^T)$.

Recall that K^* denotes the mirror image of K, and M_K denotes the Seifert matrix of K.

Exercise 1 Prove that $M_{K^*} = -M_K^T$. Conclude that if K is a knot, then $\Delta_{K^*} = \Delta_K$ (so the Alexander polynomial cannot distinguish a knot from its mirror image).

Exercise 2 Let $A \sim_S B$. Prove that $\sigma(A + A^T) = \sigma(B + B^T)$ and $n(A + A^T) = n(B + B^T)$. Conclude that the signature and nullity are link invariants.

Exercise 3 Calculate the signature of the two trefoils, and of the figure-8 knot.

Exercise 4 Prove that $\sigma_{K^*} = -\sigma_K$ for every link.

Exercise 5 Prove that if K is a knot, then $n_K = 0$ and σ_K is an even number.