Jones Polynomial with torus knots

Definition 1: Recall the properties for Kaufmann bracket polynomial $P(A)$. Note that we are now modulo regular equivalecne, so we cannot untwist.

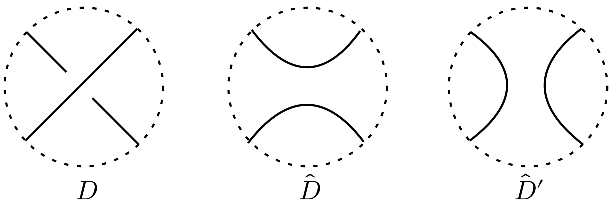
- For trivial diagram we have

$$P_D(A) = 1$$

- For $D = D_1 \sqcup D_2$ we have

$$P_D(A) = -(A^2 + A^{-2}) P_{D_1}(A) P_{D_2}(A)$$

- And in general we have the following skein relation.



$$P_D(A) = A P_{\hat{D}} + A^{-1} P_{\hat{D}'}$$

- From these we have for diagram D_n of trivial knot with n twists

$$P_{D_n}(A) = A^{\pm 3n}$$

Definition 2: Recall that Jones polynomial can be calculated as

$$V_D(t) = (-t^{\frac{3}{4}})^{w(D)} P_D(t^{-\frac{1}{4}})$$

where $w(D)$ is the writh (sum of crossing numbers in all crossings).

Exercise 1: Show that for torus knots we have $K(q,r) = K(r,q)$.

Exercise 2: The general formula for calculating Jones polynomial for torus knot $K(q,r)$ is

$$V_{K(q,r)}(t) = t^{(q-1)(r-1)/2} \frac{1 - t^{q+1} - t^{r+1} + t^{r+q}}{1 - t^2}$$

- Convince yourself that the formula is really a polynomial when $K(q,r)$ is a knot.
- Using this fact and the fact that figure eight knot has Jones polynomial equal to $t^2 + t^{-2} - t - t^{-1} + 1$ show that figure eight is not a torus knot.
- Prove the formula for $r = 2$.