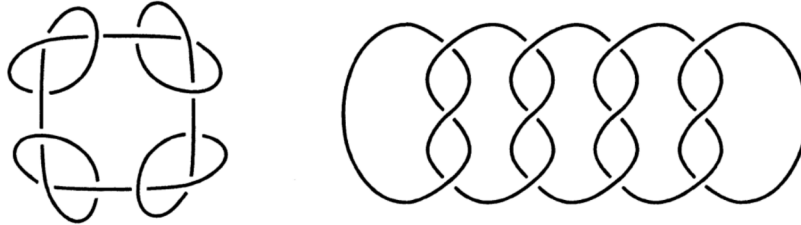# Knot decompositions

Exercise 1:   Show that the following knots are equivalent.

Exercise 2:   Take you favourite knot that is not prime. Find a diagram for it, which cannot be split into two knots.

Exercise 3:   Decompose given knots (physical strings at the lecture) into prime knots, are they equivalent?

Definition: We define the *crossing number $c(K)$* for a knot K as the minimum number of crossings in a diagram for K.

Exercise 4:   Show that there are no nontrivial knots with crossing number $c(K) < 3$.

Exercise 5:   Show that trefoils (left and right handed) are the only knots with crossing number 3.

Definition: We have a magic wand, that we can aim at any intersection on a diagram of a knot and it switches under-over crossings. We define *unknotting number* of a knot the minimum number of times we have to use the wand to untie the knot.

Exercise 6:   What is the unknotting number for trifoil?

Exercise 7:   Show that the following knot has unknotting number at most 3.