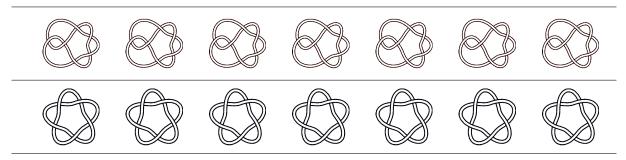
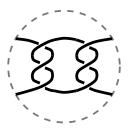
## Knot colorings

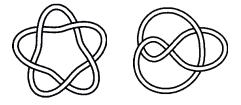
Exercise 1: Distinguish the following two knots with themselfs and with the unknot.



**Exercise 2:** Suppose a knot diagram D contains a portion like the one shown below. Show that the knot represented by D cannot be the unknot. **Bonus**: Show that it could be a trefoil.

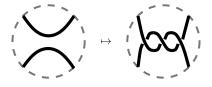


**Exercise 3:** Show that the following knots cannot be distinguished by *p*-coloring invariants.



**Definition:** Let  $col_3(K)$  be the number of 3-colorings of knot K.

**Exercise 4:** Show that if a link L is changed into a new link  $L_0$  by the local insertion of three half-twists as shown, then  $\operatorname{col}_3(L) = \operatorname{col}_3(L_0)$ .



**Exercise 5\*:** Show that  $\operatorname{col}_3(K_1 \# K_2) = \frac{1}{3} \operatorname{col}_3(K_1) \operatorname{col}_3(K_2)$