## 6. cvičení z Teorie reprezentací (28. května)

**Příklad 0:**

$$Q = \overset{1}{\circ} \underset{\beta}{\overset{\alpha}{\rightrightarrows}} \overset{2}{\circ}$$

$$X = k^2 \underset{\begin{pmatrix}0&0\\1&0\\0&1\end{pmatrix}}{\overset{\begin{pmatrix}1&0\\0&1\\0&0\end{pmatrix}}{\rightrightarrows}} k^3$$

V toulci $Q$ je jediný ~~stok~~, a to je vrchol 2.
My budeme počítat reflexi $X$ vzhledem k tomu vrcholu 2.

Toulec $\sigma_2 Q$ vypadá: $\sigma_2 Q = \overset{1}{\circ} \underset{\beta}{\overset{\alpha}{\leftleftarrows}} \overset{2}{\circ}$.

Jaký $\underline{\dim} X = (2,3)^T$. Jaká je reflexe?

$$\sigma_2(\underline{\dim} X) = \underline{\dim} X - \frac{2(\underline{\dim} X, e_2)}{(e_2, e_2)} e_2$$

$$q(\underline{\dim} X + e_2) = q((2,4)^T) =$$
$$= 2^2 + 4^2 - 2\cdot 4 - 2 \cdot 4 = 20 - 16 = 4$$

$$q(\underline{\dim} X) = q((2,3)^T) =$$
$$= 2^2 + 3^2 - 2\cdot 3 - 2\cdot 3 = 13 - 12 = 1$$

$$q(e_2) = q((0,1)^T) =$$
$$= 1$$

$$\Rightarrow (\underline{\dim} X, e_i) = 4 - 1 - 1 = 2$$

$$(e_i, e_i) = 2 \quad (q(2e_i) = 4)$$

$$\Rightarrow \sigma_i((2,3)^T) = (2,3)^T - \frac{2\cdot 2}{2}(0,1)^T$$

$$= (2,1)^T$$

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$X_\alpha \quad X_\beta$

$$X_1 \oplus X_1 \xrightarrow{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} X_2$$

Toto zobrazení je epi. Tedy jádro má dimenzi

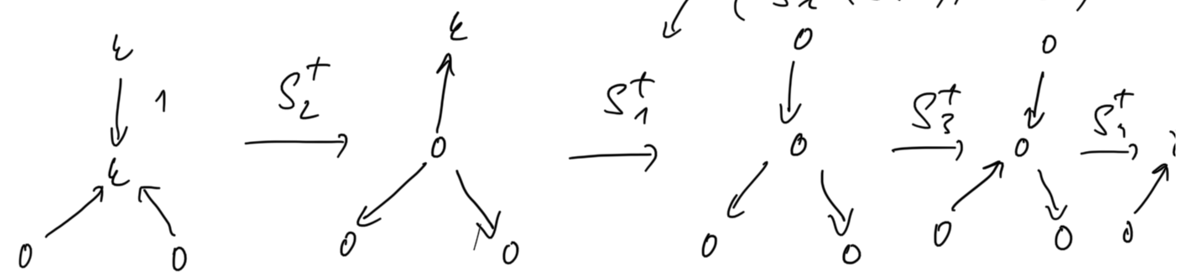
$$\to \ker(X_\alpha \mid X_\beta) \hookrightarrow \overset{\alpha}{X_1} \oplus \overset{\beta}{X_1} \qquad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$k \xhookrightarrow{\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}} k^2 \oplus k^2 \qquad \alpha, \beta \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad S_2^+(X) = k^2 \underset{\begin{pmatrix} -1 \\ 0 \end{pmatrix}}{\overset{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\leftleftarrows}} k$$

$$S_2^+(X) = X_1 \leftleftarrows \ker(X_\alpha \mid X_\beta)$$

<u>Příklad 1</u>:




$$(1,1,0,0)^T \xrightarrow{\sigma_2} (1,0,0,0)^T \xrightarrow{\sigma_1} (-1,0,0,0)^T \xrightarrow{\sigma_3} (\;) \xrightarrow{\sigma_1} (\;$$

$$\overset{\shortparallel}{x}$$

$$\sigma_1(e_1) = e_1 - \frac{2(e_1 \ldots}{(e_1, \ldots}$$

$$\sigma_2(x) = x - \frac{2(x, e_2)}{(e_2, e_2)} e_2 \qquad = e_1 - 2e_1 \ldots$$

$$= -e_1$$

$$(x, e_2) = q(x + e_2) - q(x) - q(e_2)$$

$$q(x + e_2) = q((1,2,0,0)^T) = 1^2 + 2^2 - 2 \cdot 1 = 3$$

$$q(x) = q((1,1,0,0)^T) = 1^2 + 1^2 - 1 \cdot 1 = 1$$

$q(e_2) = 1 \quad q((0,1,0,0)^T) = 1^2 = 1$

$(e_2, e_2) = 2$

$$\frac{2(3-1-1)}{2} = 1$$

$\Rightarrow \quad \sigma_2(x) = (1,0,0,0)$.

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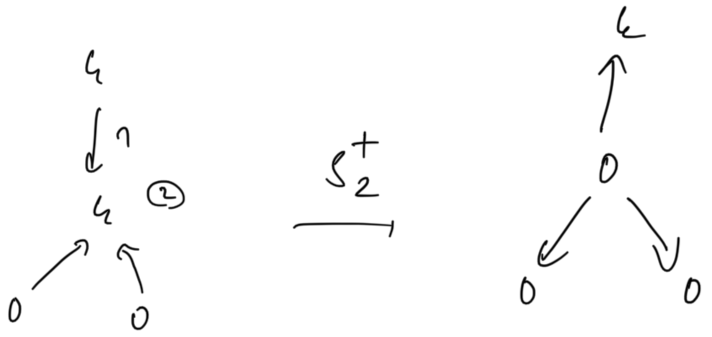

$(e_i, e_i) \neq 2$

$q(u e_i) = \sum_{j \in Q_0} u^2 (e_i)_j^2 - \sum_{\alpha: j \to k \in Q_1} u^2 (e_i)_j \ldots = u^2$

$(e_i, e_i) \underset{= 2e_i}{=} q(e_i + e_i) - 2q(e_i)$

$0 \longrightarrow k \xrightarrow{1} k$

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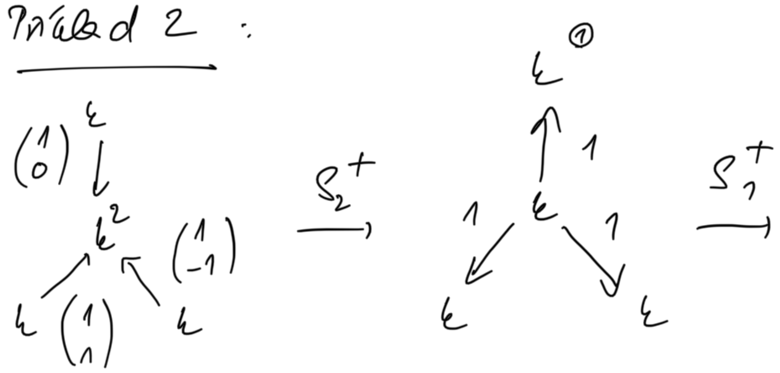
Příklad 2:



$(1,2,1,1)^T \xrightarrow{\sigma_2} (1,1,1,1)^T \xrightarrow{\sigma_1}$

$$k \oplus k \oplus k \xrightarrow{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}} k^2$$

zajímavé má jádro

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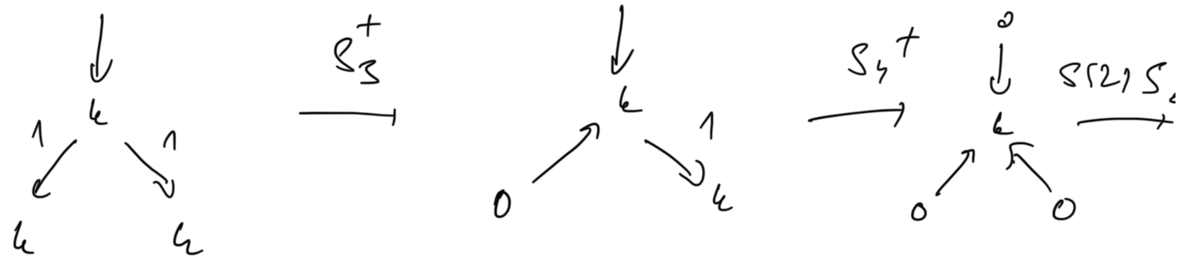


$(0,1,1,1,1) \xmapsto{\sigma_3}$