NMAG442 Representation Theory of Finite-Dimensional Algebras

Excercise session 4—April 7, 2022

Our goal today is to continue exploring path algebras and their representations and to employ some previously discussed concepts in doing so.

We work over an algebraically closed field k and with finite-dimensional modules.

Path algebras and their representations

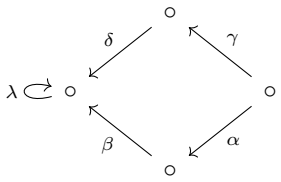
Exercise 1 (Kronecker quiver). For any $n \in \mathbb{N}$ find φ, ψ maps such that

$$k^n \underset{\psi}{\overset{\varphi}{\rightrightarrows}} k^{n+1}$$

is indecomposable. Compute the endomorphism ring of such a representation.

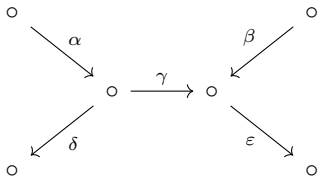
Exercise 2 (Exercise 5 in II.4 in [1]). Let Q be a finite and acyclic quiver. Prove that kQ is a connected $k-$algebra if and only if kQ/R_Q^2 is a connected $k-$algebra (R_Q is the ideal of kQ generated by all arrows of Q).

Exercise 3 (Examples 5(d) and 7(d) in III.2 in [1]). Given a quiver on four vertices:



bound by the following relations $\alpha\beta = \gamma\delta$ and $\lambda^3 = 0$ (observe that the ideal generated by these relations is admissible), compute all simple, indecomposable projective (with radicals) and injective modules (with factors by socles) over this quiver.

Exercise 4. Given a quiver on six vertices:



bound by the relation $\alpha\gamma\varepsilon = 0$ (observe that the ideal generated by this relation is admissible), compute all simple, indecomposable projective (with radicals) and injective modules (with factors by socles) over this quiver.

Exercise 5 (After Exercise 16 in II.4 in [1]). Show that $\mathbb{C}$ is not isomorphic to $\mathbb{R}Q/I$ for any quiver Q and I admissible ideal of $\mathbb{R}Q$; although, $\mathbb{C}$ is finite-dimensional, basic, and connected algebra over $\mathbb{R}$.

References

[1] Assem, I., Skowronski, A., and Simson, D. *Elements of the Representation Theory of Associative Algebras: Volume 1: Techniques of Representation Theory*, vol. 65. Cambridge University Press, 2006.

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