NMAG442 Representation Theory of Finite-Dimensional Algebras

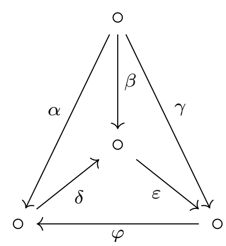
Excercise session 5—April 21, 2022

Our goal today is to finish exploring path algebras and their representations and to complete the exercises from the last session. We may also get to some hereditary algebras.

We work over an algebraically closed field k and with finite-dimensional modules.

Path algebras and their representations

Exercise 1. Given a quiver on four vertices:



bound by the relation $\delta\epsilon\varphi = 0$ (observe that the ideal generated by this relation is admissible), compute all simple, indecomposable projective (with radicals) and injective modules (with factors by socles) over this quiver.

Exercise 2 (Kronecker quiver). For any $n \in \mathbb{N}$ find φ, ψ maps such that

$$k^n \underset{\psi}{\overset{\varphi}{\rightrightarrows}} k^{n+1}$$

is indecomposable. Compute the endomorphism ring of such a representation.

Exercise 3 (Exercise 5 in II.4 in [1]). Let Q be a finite and acyclic quiver. Prove that kQ is a connected $k-$algebra if and only if kQ/R_Q^2 is a connected $k-$algebra (R_Q is the ideal of kQ generated by all arrows of Q).

Exercise 4 (After Exercise 16 in II.4 in [1]). Show that $\mathbb{C}$ is not isomorphic to $\mathbb{R}Q/I$ for any quiver Q and I admissible ideal of $\mathbb{R}Q$; although, $\mathbb{C}$ is finite-dimensional, basic, and connected algebra over $\mathbb{R}$.

Hereditary algebras

Exercise 5 (Exercise 1 in VII.6 in [1]). Show that the following matrix algebras are hereditary:

(i) $$\begin{bmatrix} k & 0 & 0 & 0 \\ k & k & 0 & 0 \\ k & k & k & 0 \\ k & k & 0 & k \end{bmatrix}$$

(ii) $\begin{bmatrix} k & 0 & 0 & 0 \\ k & k & 0 & 0 \\ k & 0 & k & 0 \\ k & 0 & 0 & k \end{bmatrix}$

(iii) $\begin{bmatrix} k & 0 & 0 & 0 & 0 & 0 \\ k & k & 0 & 0 & 0 & 0 \\ k & 0 & k & 0 & 0 & 0 \\ k & 0 & k & k & 0 & 0 \\ k & 0 & k & 0 & k & 0 \\ k & 0 & k & 0 & k & k \end{bmatrix}$

References

[1] Assem, I., Skowronski, A., and Simson, D. *Elements of the Representation Theory of Associative Algebras: Volume 1: Techniques of Representation Theory*, vol. 65. Cambridge University Press, 2006.

Feel free to reach me at `jakub.kopriva@mff.cuni.cz`. Also, I am available for short consultations on problems from the exercise sessions after previous arrangment via e-mail.